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Challenges in MC Simulations: pp vs e^+e^-

Stefan Höche, Alan Price

Fermi National Accelerator Laboratory

U.S. FCC Hackathon

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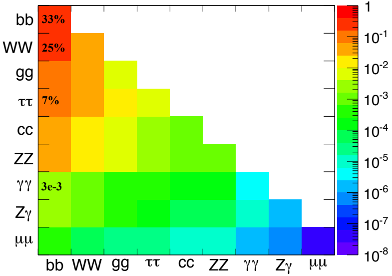
Outline

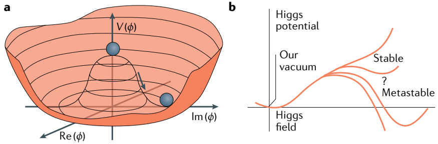
- A quick tour of LHC simulations
  - Big picture
  - Theory challenges
  - Lessons for FCC
- A tour of current FCC simulations
  - Big picture
  - Theory challenges
  - Lessons from LEP
- Towards higher precision
  - Perturbative QCD
  - QED / EW
  - Computing
- Needs and requirements

LHC – What we are preparing for

- Higgs self interaction is key to understanding of EW sector
- Measurement will require careful combination of many analyses with full HL-LHC data set
- Heavy flavor channels needed for high statistical significance



[J. Alison] LHCP ’24



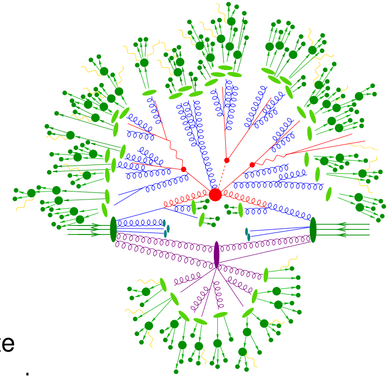
[Bass, DeRoeck, Kado] Nat. Rev. Phys. 3 (2021) 608

- Predictions for heavy quark production as part of inclusive heavy plus light flavor jets difficult to obtain at high precision
- Precise extraction of / limit setting on triple Higgs coupling depends crucially on understanding of all final states

Schematics of LHC simulations

Need to cover large dynamic range

- Short distance interactions
  - Signal process
  - Radiative corrections
- Long-distance interactions
  - Hadronization
  - Particle decays



Divide and Conquer

- Quantity of interest: Total interaction rate
- Convolution of short & long distance physics

$$\sigma_{p_1 p_2 \to X} = \sum_{i,j \in \{q,g\}} \int \mathrm{d}x_1 \mathrm{d}x_2 \underbrace{f_{p_1,i}(x_1, \mu_F^2) f_{p_2,j}(x_2, \mu_F^2)}_{\text{long distance}} \underbrace{\hat{\sigma}_{ij \to X}(x_1 x_2, \mu_F^2)}_{\text{short distance}}$$

QCD theory as the primary tool

- $\hat{\sigma}_{ij\to n}(\mu_F^2) \to$ Collinearly factorized fixed-order result at N^{x}LO

  Implemented in fully differential form to be maximally useful

  Tree level: $\mathrm{d}\Phi_n \; B_n$

  - Automated ME generators + phase-space integrators

  1-Loop level: $\mathrm{d}\Phi_n \left(B_n + V_n + \sum C + \sum I_n\right) + \mathrm{d}\Phi_{n+1}\left(R_n - \sum S_n\right)$

  - Automated loop ME generators + integral libraries + IR subtraction

  2-Loop level: It depends ...

  - Individual solutions based on SCET, q_T subtraction, P2B

- $f_i(x, \mu_F^2) \to$ Collinearly factorized PDF at N^{y}LO

  Evaluated at $O(1\mathrm{GeV}^2)$ and expanded into a series above $1\mathrm{GeV}^2$

  DGLAP: $\frac{\mathrm{d}x\, x f_a(x,t)}{\mathrm{d}\ln t} = \sum_{b=q,g} \int_0^1 \mathrm{d}\tau \int_0^1 \mathrm{d}z\, \frac{\alpha_s}{2\pi} \left[z P_{ab}(z)\right]_+ \tau f_b(\tau, t)\, \delta(x - \tau z)$

  - Parton showers, dipole showers, antenna showers, ...

  Matching: $\mathrm{d}\Phi_n \, \frac{S_n}{B_n} \leftrightarrow \frac{\mathrm{d}t}{t} \mathrm{d}z \, \frac{\alpha_s}{2\pi} \, P_{ab}(z)$

  - MC@NLO, POWHEG, Geneva, MINNLO$_{\mathrm{PS}}$, ...

Directions of development

Much effort focused on perturbative QCD

- Phenomenologically interesting: Drives jet production, b-tagging, ...
- Experimentally relevant: Often source of largest uncertainty

Fixed-order aspects

- (N)NLO fixed order QCD
- Matching to parton shower
- Combination with QED (YFS)
- ... and NLO EW corrections

All-order aspects

- (N)NLL precision
- Heavy quark effects
- Sub-leading color & spin
- Threshold effects

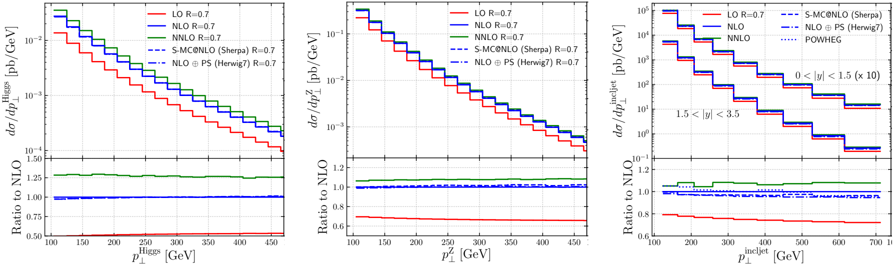
Understanding uncertainties & limitations

- Multi-year projects in context of LesHouches workshops to compare different generators on equal footing
- Growing community of MC devs & expert users in experiments with ties to MC groups & knowledge of common pitfalls in MC usage

Uncertainties in QCD NLO+PS matching

[Bellm at al.] arXiv:1903.12563

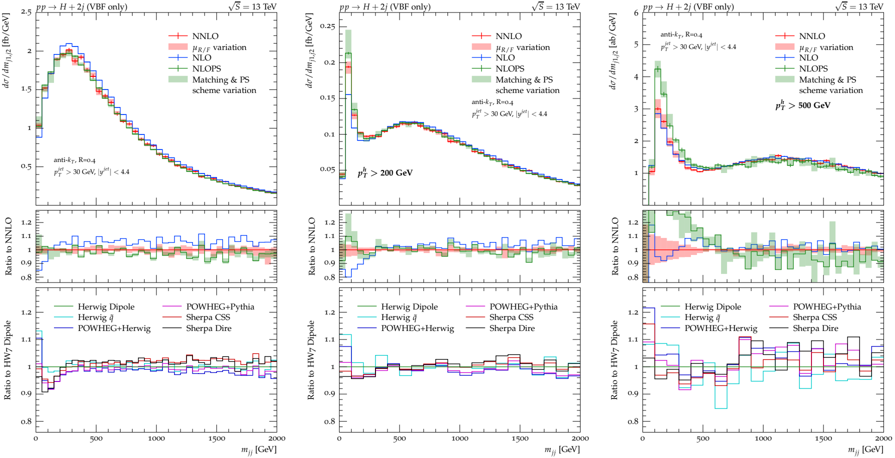
- Ratio of inclusive jet-$p_\perp$ cross sections for different radii in $pp \to jets$



Uncertainties in QCD NLO+PS matching

[Buckley et al.] arXiv:2105.11399

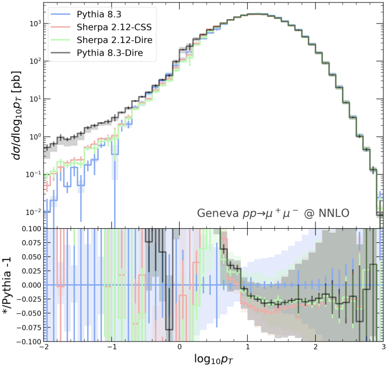
- m_{jj} of two leading jets in VBF Higgs production

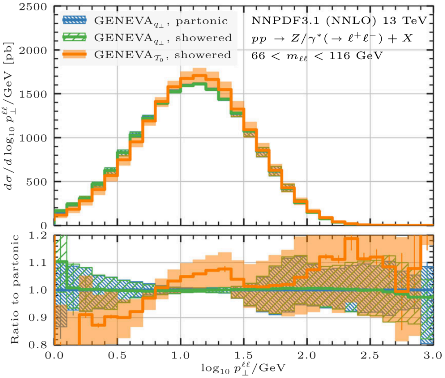


Uncertainties in QCD NNLO+PS matching

[D. Napoletano, HP2 2022], [Alioli et al.] arXiv:2102.08390

- NNLO+PS precise predictions for $pp \to Z$ from Geneva
- Matched to shower by vetoing events with $r_N(\Phi_{N+M}) > r_N$
- Significant residual uncertainties, even though NNLO

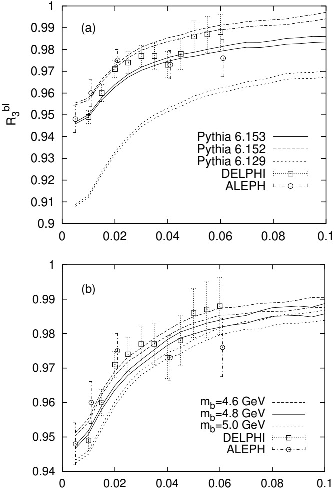




- Parton shower scheme uncertainty
- Choice of resolution variable

Heavy quark production

- Both high-energy limit and threshold region should be modeled as well as possible, but
- Infrared finite prediction for $g \to Q\bar{Q}$ leaves splitting functions somewhat arbitrary
- Soft gluon emission off light/heavy quarks associated with $\alpha_s(k_T^2)$, i.e. "correct" scale is k_T^2 [Amati et al.] NPB173(1980)429, but no such argument to set scale for $g \to Q\bar{Q}$
  $\to$ HQ production rate not very stable w.r.t. parton shower variations
- A number of different prescriptions, e.g.
  [Norrbin,Sjöstrand], hep-ph/0010012,
  [Gieseke,Stephens,Webber] hep-ph/0310083,
  [Schumann,Krauss] arXiv:0709.1027,
  [Gehrmann-deRidder,Ritzmann,Skands] arXiv:1108.6172
  varying success in describing expt. data



[Norrbin,Sjöstrand] hep-ph/0010021

Heavy quark production

- Two different approaches to dealing with heavy-quark masses:
  - 4-flavor scheme (4FS): Decoupling scheme - (no b-quarks in PDF)
  - 5-flavor scheme (5FS): Minimal subtraction scheme
- Calculations can be matched by
  - Re-expressing both in same renormalization scheme
  - Subtracting the overlap

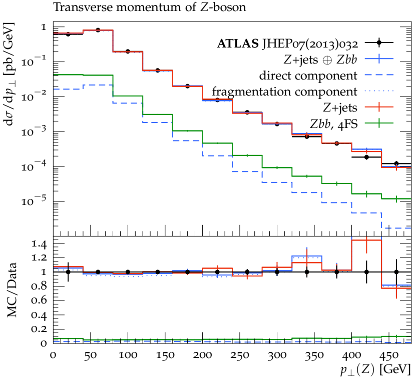
$$\sigma^{\text{FONLL}} = \sigma^{\text{massive}} + (\sigma^{\text{massless}} - \sigma^{\text{massive, 0}})$$

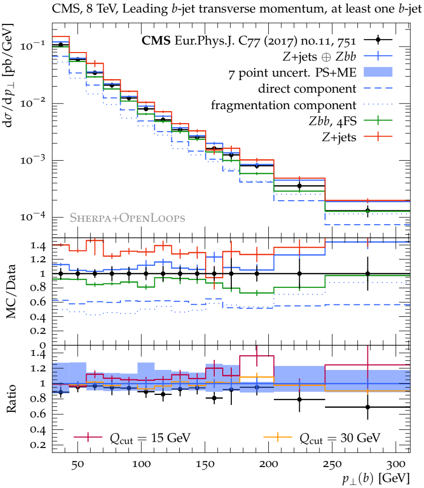
- This has been applied extensively to inclusive observables and is know as fixed-order next-to-leading log (FONLL) scheme
  [Cacciari,Frixione,Mangano,Nason,Ridolfi] hep-ph/0312132,
  [Forte,Napoletano,Ubiali] arXiv:1508.01529, arXiv:1607.00389, ...
- Extension to differential observables is needed for MC simulations
  $\rightarrow$ fully differential "fusing" algorithm [Krause,Siegert,SH] arXiv:1904.09382

Heavy quark production

- Z+jets vs $Zb\bar{b}$ at LHC

[Krause,Siegert,SH] arXiv:1904.09382





	Data [pb]	**Fusing** [pb]
$Z+\geq 1b$	$3.55 \pm 0.24_{\text{comb}}$	$3.80(5) \pm ^{0.83}_{0.33}$
$Z+\geq 2b$	$0.331 \pm 0.037_{\text{comb}}$	$0.282(4) \pm ^{0.027}_{0.022}$

Improvements needed for FCC

Fully differential high precision calculations

- NNLO QCD subtraction formalism
- Mixed QCD/EW corrections

Resummation and matching to fixed order

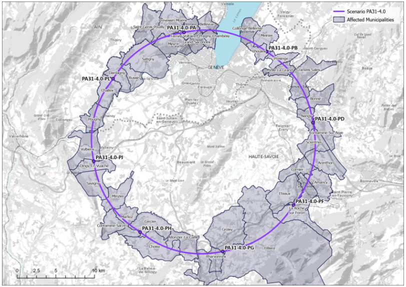
- Parton showers at NNLL precision
- Reduction of matching scheme uncertainty

Incorporation of quark mass effects

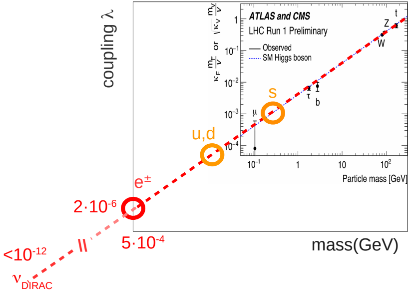
- Formal accuracy beyond FONLL-A
- Interplay with fragmentation functions

FCC – What we are preparing for

- Unprecedented luminosity at Tera-Z option of a potential FCC-ee would leave no room for mis-modeling of pQCD / npQCD or QED / EW effects



[CERN] https://home.cern/science/accelerators/



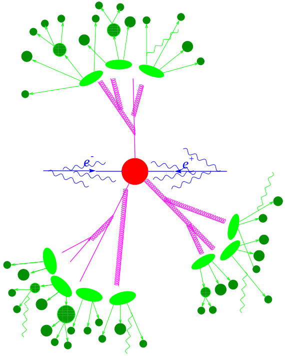
[D. d'Enterria] FCC week '24

- Extraction of Higgs Yukawa couplings would depend on precise modeling of light / heavy flavor jet production and flavor dynamics

Schematics of FCC simulations

Need to cover modest dynamic range

- Short distance interactions
  - Signal process
  - QCD radiative corrections
  - QED radiative corrections
- Long-distance interactions
  - Hadronization
  - Particle decays



Divide and Conquer

- Quantity of interest: Interaction rate
- If hadrons involved, convolution of short & long distance physics, e.g.

$$\sigma_{p_1 p_2 \to X} = \sum_{i,j\in\{q,g\}} \int \mathrm{d}x_1 \mathrm{d}x_2 \underbrace{\hat{\sigma}_{ij+X}(x_1, x_2, \mu_F^2)}_{\text{short distance}} \underbrace{D_{h_1,i}(x_1, \mu_F^2) D_{h_2,j}(x_2, \mu_F^2)}_{\text{long distance}} \ldots$$

Aspects of pQCD at FCC

Things to consider

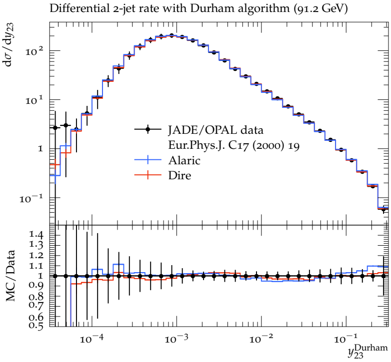
- At Tera-Z, the dynamic range is modest: $\sqrt{s} \approx 20 \times m_b$
  QCD radiative effects are important, but still limited
  We get about 7 gluons on average before hadronization
- This implies that understanding sub-leading powers
  is more important for precision than controlling higher logs
- Parton showers include some of those effects through
  exact phase-space & scalar splitting functions ($\nearrow$ later)

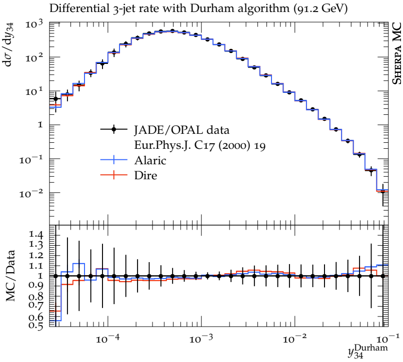
Consequences for MC development

- Parton-showers have to satisfy boundary conditions
  from analytic resummation, but we need to go beyond
- Much can be done by matching to fixed order, because the
  average number of emissions between $\sqrt{s}$ and $\Lambda_{\rm QCD}$ is small

Typical performance of parton-showers

[Herren,Krauss,Reichelt,Schönherr,SH] arXiv:2208.06057





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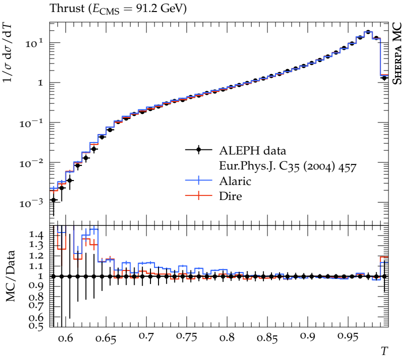
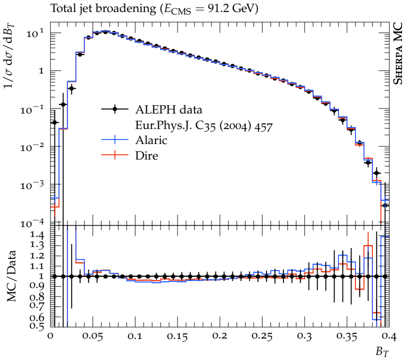
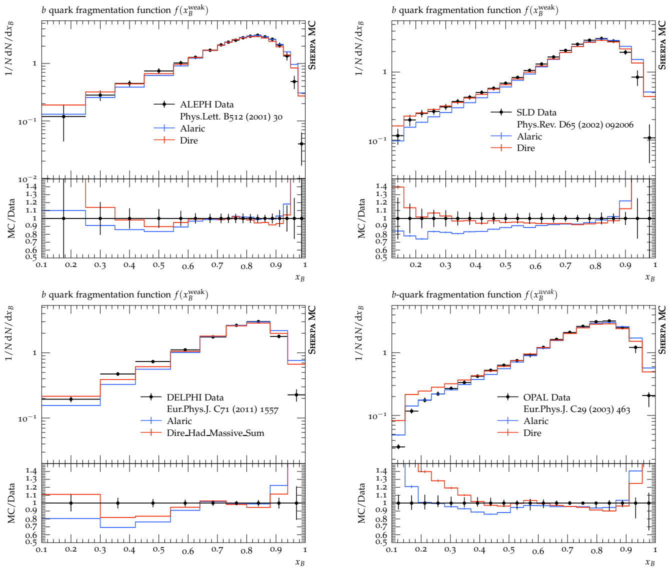




Typical performance in heavy quark evolution

[Assi,SH] arXiv:2307.00728



Impact of the momentum mapping

[Dasgupta,Dreyer,Hamilton,Monni,Salam] arXiv:1805.09327

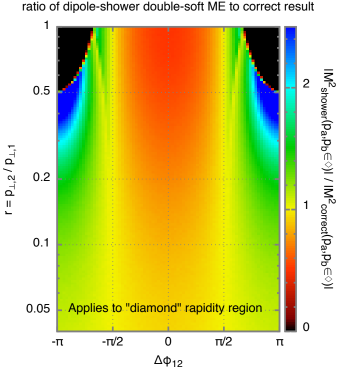
- Some dipole-like momentum mappings violate strong ordering approximation

$$p_k^\mu = \left(1 - \frac{p_{ij}^2}{2\tilde{p}_{ij}\tilde{p}_k}\right)\tilde{p}_k^\mu$$

$$p_i^\mu = \tilde{z}\,\tilde{p}_{ij}^\mu + (1-\tilde{z})\frac{p_{ij}^2}{2\tilde{p}_{ij}\tilde{p}_k}\tilde{p}_k^\mu + k_\perp^\mu$$

$$p_j^\mu = (1-\tilde{z})\,\tilde{p}_{ij}^\mu + \tilde{z}\frac{p_{ij}^2}{2\tilde{p}_{ij}\tilde{p}_k}\tilde{p}_k^\mu - k_\perp^\mu$$

- Angular correlations across multiple emissions due to recoil on splitter in anti-collinear region
- Spoils $\alpha_s \to 0$ consistency check $\leftrightarrow$ NLL accuracy cannot be achieved



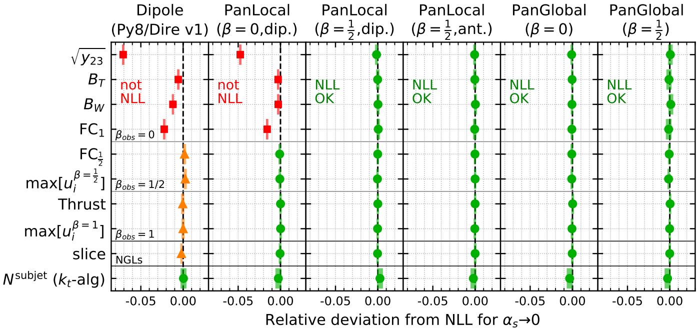
Impact of the momentum mapping

[Dasgupta,Dreyer,Hamilton,Monni,Salam,Soyez] arXiv:2002.11114

- Problem can be solved e.g. by partitioning of antenna radiation pattern and choosing a suitable evolution variable ($\beta \sim 1/2$)

$$k_T = \rho v e^{\beta|\bar{\eta}|} \qquad \rho = \left(\frac{s_i s_j}{Q^2 s_{ij}}\right)^{\beta/2}$$

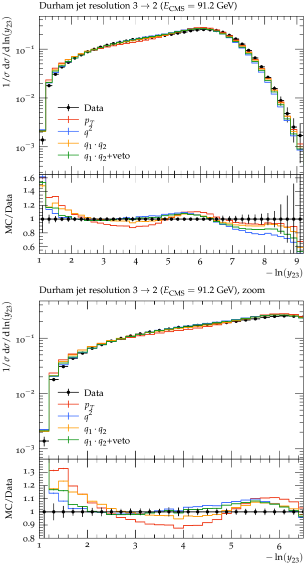
- NLL correct for global and non-global observables in $e^+e^- \to$hadrons



Impact of the momentum mapping

[Bewick,Ferrario-Ravasio,Richardson,Seymour] arXiv:1904.11866

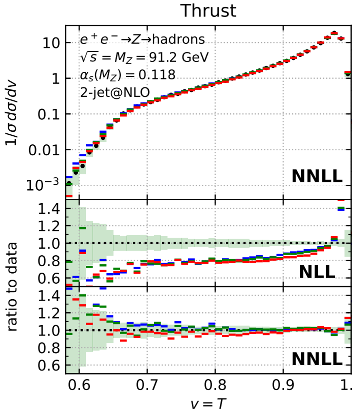
- Note: Recoil schemes affect logarithmic accuracy but impact also phase-space coverage & sub-leading power effects
- In context of angular ordered Herwig 7 (NLL accurate for global observables)
  - q_T preserving scheme:
    Maintains logarithmic accuracy
    Overpopulates hard region
  - q^2 preserving scheme:
    Breaks logarithmic accuracy
    Good description of hard region
  - Dot product preserving scheme (new):
    Maintains logarithmic accuracy
    Good description of hard radiation

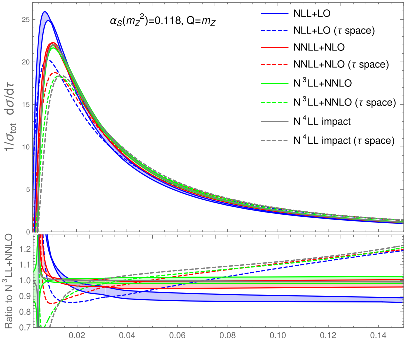


NNLL evolution for event shapes

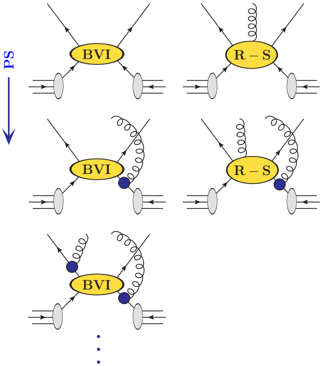
[vanBeekveld et al.] arXiv:2406.02661

- NNLL for global event shapes achieved recently
- Found differences of $\mathcal{O}(20\%)$ between NLL and NNLL
  Compare to analytic computation [Aglietti,Ferrera,Ju,Miao] arXiv:2502.01570
- Better understanding needed to achieve target precision for FCC-ee
  $\rightarrow$ Is there a need for N^{3}LL, or rather sub-leading power?



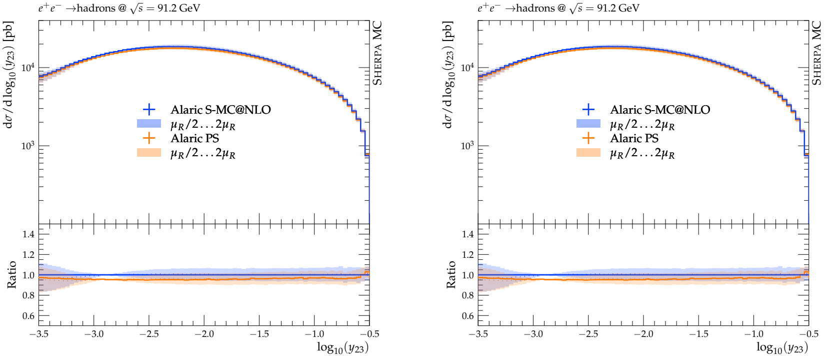


MC@NLO matching



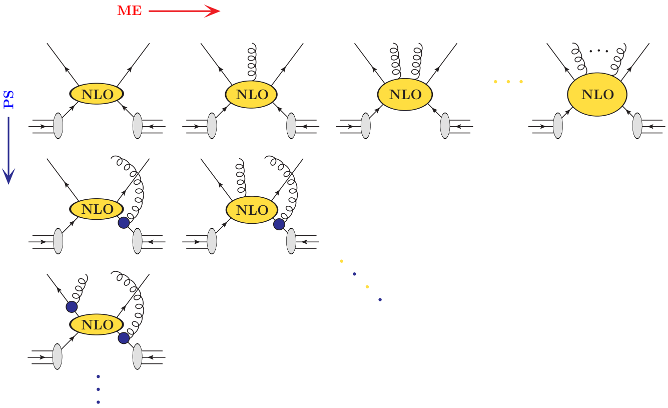
Typical performance of MC@NLO matching

[Krauss,Meinzinger,Reichelt,SH] arXiv:2507.22837



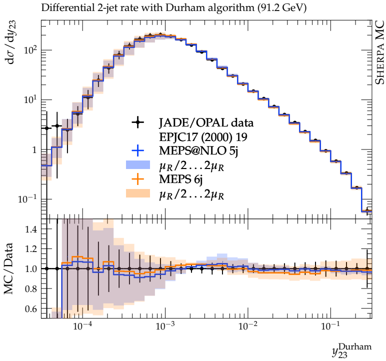
- Jet rates in Durham algorithm
- Radiation pattern determined almost exclusively by PS

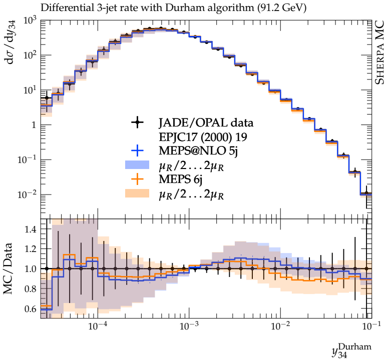
Multi-jet merging



Typical performance of matching & merging

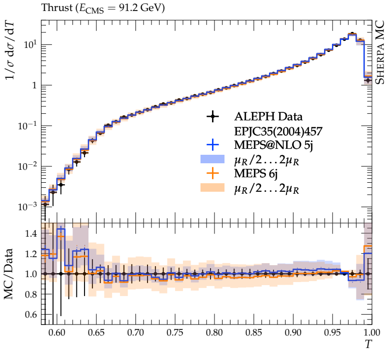
[Krauss,Meinzinger,Reichelt,SH] arXiv:2507.22837

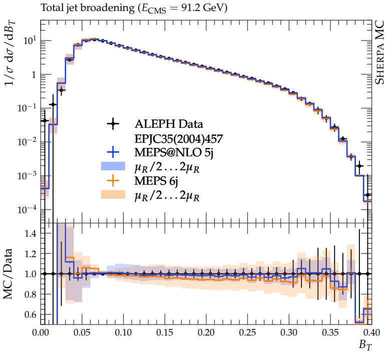




Typical performance of matching & merging

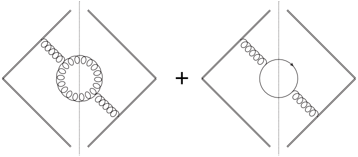
[Krauss,Meinzinger,Reichelt,SH] arXiv:2507.22837





Towards NLO QCD evolution: Soft limit

- Approximate soft-gluon emission times collinear decay in $q(i)\bar{q}(j)g(1)g(2)$ using semi-classical limit and gluon splitting function



$$= \sum_{b=q,g} j_{ij,\mu}(p_{12}) j_{ij,\nu}(p_{12}) \frac{P_{gb}^{\mu\nu}(z_1)}{s_{12}}$$

$$P_{gq}^{\mu\nu}(z) = T_R\left(-g^{\mu\nu} + 4\,z(1-z)\,\frac{k_\perp^\mu k_\perp^\nu}{k_\perp^2}\right)$$

$$P_{gg}^{\mu\nu}(z) = C_A\left(-g^{\mu\nu}\left(\frac{z}{1-z} + \frac{1-z}{z}\right) - 2\,(1-\varepsilon)z(1-z)\,\frac{k_\perp^\mu k_\perp^\nu}{k_\perp^2}\right)$$

- Combine with phase space for one parton emission in collinear limit $D = 4 - 2\varepsilon$, $y = s_{12}/Q^2$, see for example [Catani,Seymour] hep-ph/9605323

$$\mathrm{d}\Phi_{+1} = \frac{Q^{2-2\varepsilon}}{16\pi^2}\frac{(4\pi)^\varepsilon}{\Gamma(1-\varepsilon)}\,\mathrm{d}y\,\mathrm{d}z\,\left[y\,z(1-z)\right]^{-\varepsilon}$$

- Perform Laurent series expansion

$$\frac{1}{y^{1+\varepsilon}} = -\frac{\delta(y)}{\varepsilon} + \sum_{n=0}^{\infty}\frac{\varepsilon^n}{n!}\left(\frac{\ln^n y}{y}\right)_+$$

Towards NLO QCD evolution: Soft limit

- $\mathcal{O}(\varepsilon^0)$ differential remainder terms have contributions proportional to

$$g \to q\bar{q}: \quad T_R\left[2z(1-z) + \big(1-2z(1-z)\big)\ln(z(1-z))\right]$$

$$g \to gg: \quad 2C_A\left[\frac{\ln z}{1-z} + \frac{\ln(1-z)}{z} + \big(-2+z(1-z)\big)\ln(z(1-z))\right]$$

- Integration over z, addition of some semi-classical terms & one-loop soft current gives two-loop cusp anomalous dimension

$$K = \left(\frac{67}{18} - \frac{\pi^2}{6}\right)C_A - \frac{10}{9}T_R n_f$$

  - Local K-factor for soft-gluon emission
  - Scheme dependent: originates in dim. reg. and $\overline{\mathrm{MS}}$
  - **Can be absorbed in effective coupling** [Catani,Marchesini,Webber] NPB349(1991)635

- Similarly, we find $\mathcal{O}(\varepsilon^0)$ contributions proportional to

$$\frac{\alpha_s}{2\pi}\beta_0 \log\frac{(p_i p_{12})(p_{12} p_j)}{(p_i p_j)\mu^2}$$

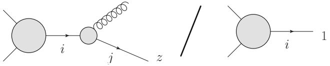
  - Can be eliminated by setting scale to transverse mass of soft pair
  - **Leading NLO correction** [Amati, et al.] NPB173(1980)429

Towards NLO QCD evolution: Collinear limit

- Higher-order DGLAP evolution kernels from factorization
  [Curci,Furmanski,Petronzio] NPB175(1980)27, [Floratos,Kounnas,Lacaze] NPB192(1981)417

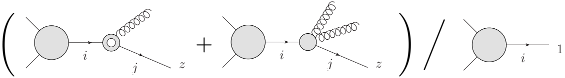
$$D^{(0)}_{ji}(z,\mu) = \delta_{ij}\delta(1-z) \qquad \leftrightarrow$$

$$D^{(1)}_{ji}(z,\mu) = -\frac{1}{\varepsilon}P^{(0)}_{ji}(z) \qquad \leftrightarrow$$



$$D^{(2)}_{ji}(z,\mu) = -\frac{1}{2\varepsilon}P^{(1)}_{ji}(z) + \frac{\beta_0}{4\varepsilon^2}P^{(0)}_{ji}(z) + \frac{1}{2\varepsilon^2}\int_z^1 \frac{\mathrm{d}x}{x}P^{(0)}_{jk}(x)P^{(0)}_{ki}(z/x)$$

$\leftrightarrow$



- In NLO parton shower, perform computation of $P^{(1)}_{ji}$ fully differentially using modified dipole subtraction [Catani,Seymour] hep-ph/9605323

Towards NLO QCD evolution: Collinear limit

[Prestel,SH] arXiv:1705.00742

- Schematically very similar to Catani-Seymour dipole subtraction
  e.g. simplest case of flavor-changing quark splitting

$$P^{(1)}_{qq'}(z) = \mathrm{C}_{qq'}(z) + \mathrm{I}_{qq'}(z) + \int \mathrm{d}\Phi_{+1}\Big[\mathrm{R}_{qq'}(z,\Phi_{+1}) - \mathrm{S}_{qq'}(z,\Phi_{+1})\Big]$$

  - Real correction $R_{qq'}$ and subtraction terms $S_{qq'}$
    given by $1 \to 3$ splitting and factorized expression
  - Integrated subtraction term and factorization counterterm

$$\mathrm{I}_{qq'}(z) = \int \mathrm{d}\Phi_{+1} S_{qq'}(z,\Phi_{+1})$$

$$\mathrm{C}_{qq'}(z) = \int_z \frac{\mathrm{d}x}{x}\left(P^{(0)}_{qg}(x) + \varepsilon\mathcal{J}^{(1)}_{qg}(x)\right)\frac{1}{\varepsilon}P^{(0)}_{gq}(z/x)$$

$$\mathcal{J}^{(1)}_{qg}(z) = 2C_F\left(\frac{1+(1-x)^2}{x}\ln(x(1-x)) + x\right)$$

- All components of $P^{(1)}_{ij}$ eventually finite in 4 dimensions
  Can be simulated fully differentially in parton shower

Combination of soft and collinear expressions

Problems with existing splitting functions

- **Kinematical limits obscure underlying structure**
  Matching soft functions to collinear limit not straightforward
- **Different pQCD techniques for different limits**
  Soft limits in Feynman gauge, collinear ones in axial gauge

To understand the structure, we have to go back to basics
$\rightarrow$ recompute in common gauge and w/o taking limits

Say that again ... How can we NOT take limits?
It's the one thing we know how to do!

Combination of ~~soft~~ scalar and ~~collinear~~ splitting expressions

[Campbell,Knobbe,Preuss,Reichelt,SH] arXiv:2505.10408

- Gordon decomposition [Gordon] ZeitPhys140(1928)630

$$\frac{\not{p}+\not{q}}{(p+q)^2}\,T^a_{ij}\gamma^\mu = T^a_{ij}\left[\,S^\mu(p,q) + \frac{i\sigma^{\nu\mu}q_\nu}{(p+q)^2} - \frac{\gamma^\mu\not{p}}{(p+q)^2}\,\right]$$

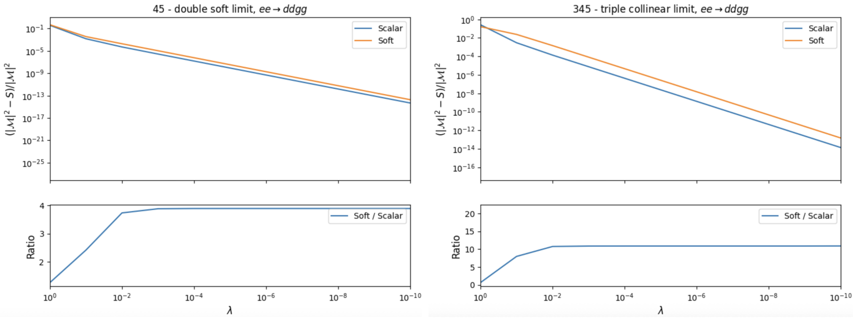
  - Leading and sub-leading (**LBK!**) soft behavior given by scalar current
    [Gell-Mann,Goldberger] PR96(1954)1433, [Brown,Goble] PR173(1968)1505

$$S^\mu(p,q) = \frac{(2p+q)^\mu}{(p+q)^2}$$

  - Magnetic term $\sigma^{\nu\mu} = i/2[\gamma^\nu,\gamma^\mu]$ due to quark spin
    $\gamma^\mu\not{p}$ generates seagull interactions of scalar theory
- Decomposition of triple & quartic gluon vertex even simpler
- Both decompositions hold at amplitude squared level [Chen et al.] arXiv:1404.5963
- **Separate scalar splitting functions & spin-dependent remainders**
  Clean identification of overlap beyond kinematical limits
- At 1-loop level, Background Field Method allows to derive
  Scalar radiators that satisfy the naive Ward identities
  $\to$ Extension of soft current [Catani,Grazzini] hep-ph/0007142

Benefits: NNLO subtraction

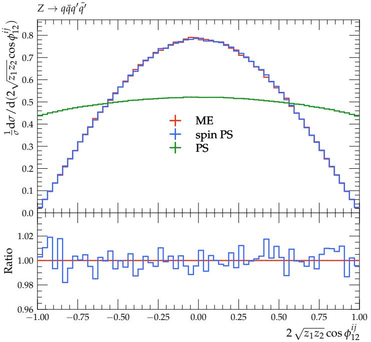
[M. Knobbe, PSR'25]



- No overlap between scalar and splitting components
  $\rightarrow$ easy assembly of complete counterterms
- Suitable for matching to NNLL parton shower

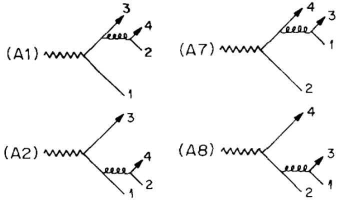
Benefits: Gluon polarization

[M. Hoppe, PSR'25]



Comparison with full, leading-colour matrix element from

R. K. Ellis, D. A. Ross, A. E. Terrano 1980

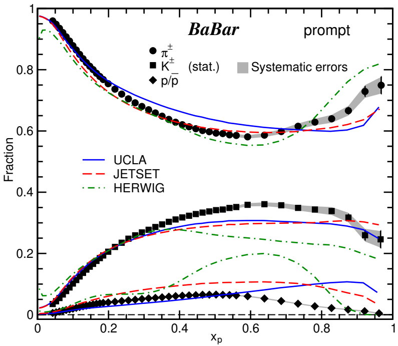


- Linear time & memory algorithm to propagate gluon polarization
- Needed for NLO evolution & NNLO matching

The Need for improved Hadronization Models

- Modeling of non-perturbative parton-to-hadron transition important for detector response, especially at low particle multiplicity
- Flavor composition of jets and identified hadron production typically challenging to model, especially at low energy
- Must be addressed in order to reach precision goals of FCC-ee

[Lees et al.] arXiv:1306.2895



The Need for improved Hadronization Models

[J. Roloff, PSR'25]

- Obvious trend in the jet response from the hadronization model
  - *Calorimeter response depends on the type of hadron, not just the energy and rapidity*
  - Using **retuned Sherpa** with LEP data on baryon and kaon fractions has a significant effect on the p_T response!

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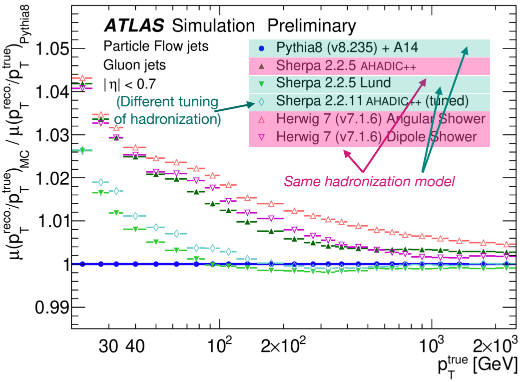
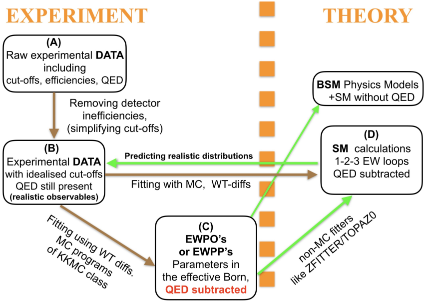


The Need for Precise QED Simulations

- Projected 2-100× improvement in measurement of EWPOs
- Permille-level uncertainties could be ignored at LEP but not at FCC-ee, particularly Tera-Z option
- QED radiative effects must be modeled as precisely as possible

[Jadach,Skrzypek] arXiv:1903.09895



Observable	Where from	Present (LEP)	FCC stat.	FCC syst	Now/FCC
M_Z [MeV]	Z linesh.	$91187.5 \pm 2.1\{0.3\}$	0.005	0.1	3
Γ_Z [MeV]	Z linesh.	$2495.2 \pm 2.1\{0.2\}$	0.008	0.1	2
$R_l^Z = \Gamma_h/\Gamma_l$	$\sigma(M_Z)$	$20.767 \pm 0.025\{0.012\}$	$6 \cdot 10^{-5}$	$1 \cdot 10^{-3}$	12
$\sigma^0_{\rm had}$[nb]	$\sigma^0_{\rm had}$	$41.541 \pm 0.037\{0.025\}$	$0.1 \cdot 10^{-3}$	$4 \cdot 10^{-3}$	6
N_ν	$\sigma(M_Z)$	$2.984 \pm 0.008\{0.006\}$	$5 \cdot 10^{-6}$	$1 \cdot 10^{-3}$	6
N_ν	$Z\gamma$	$2.69 \pm 0.15\{0.06\}$	$0.8 \cdot 10^{-3}$	$< 10^{-3}$	60
$\sin^2\theta_W^{eff} \times 10^5$	$A_{FB}^{lept.}$	$23099 \pm 53\{28\}$	0.3	0.5	55
$\sin^2\theta_W^{eff} \times 10^5$	$\langle \mathcal{P}_\tau \rangle, A_{\rm FB}^{pol,\tau}$	$23159 \pm 41\{12\}$	0.6	< 0.6	20
M_W [MeV]	ADLO	$80376 \pm 33\{6\}$	0.5	0.3	12
$A_{FB,\mu}^{M_Z \pm 3.5{\rm GeV}}$	$\frac{d\sigma}{d\cos\theta}$	$\pm 0.020\{0.001\}$	$1.0 \cdot 10^{-5}$	$0.3 \cdot 10^{-5}$	100

QED Resummation

Collinear Resummation

[Frixione et al.] JHEP03(2020)135

- Collinear logs are resummed with universal PDF
- Matched to $\mathrm{NLO}_{\mathrm{EW}}$
- Combined with Parton Shower to generate photon emissions
- Beyond NLO becomes tricky

Soft Resummation

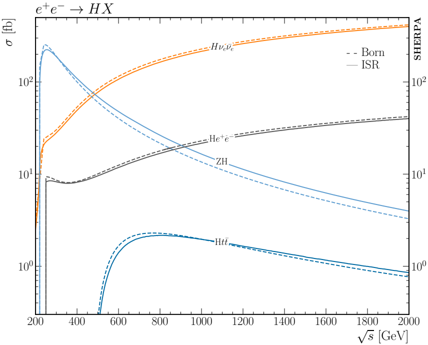
[Jadach et.al] ZPC49(1991)577, EPL17(1992)123

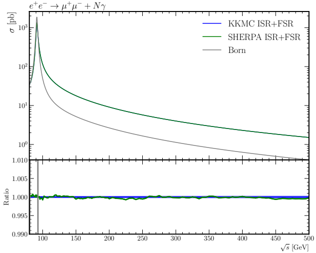
- Soft logs resummed to infinite order using the YFS method

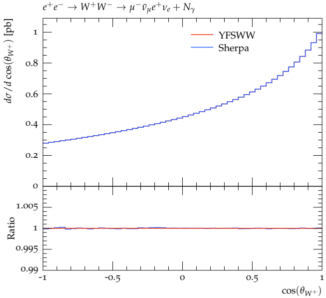
  [Yennie,Frautschi,Suura] Ann.Phys.13(1961)379
- Provides a robust scheme for the inclusion of real and virtual corrections at any order.
- Collinear terms can be added

YFS Resummation

- Modern YFS tools validated carefully against state-of-the art from LEP, e.g. KKMC [Jadach,Ward,Was] hep-ph/9912214, YFSWW [Jadach et al.] hep-ph/0104049







YFS Matching: Virtual corrections

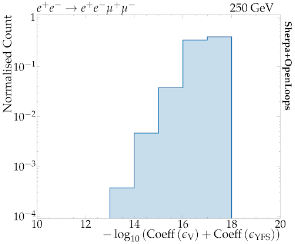
$$\tilde{\beta}_0^1(\Phi_n) = \mathscr{V}(\Phi_n) - \sum_{ij} \mathscr{D}_{ij}(\Phi_{ij})$$

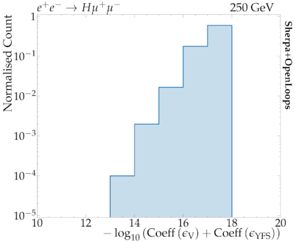
IR Finite one-loop contribution

Full One-loop amplitude

YFS Subtraction term

- Expansion of resummed soft virtual in YFS provides IR counterterms for exact virtual
- Pole cancellation occurs locally in phase space $\rightarrow$ checks against 1-loop





YFS Matching: Real corrections

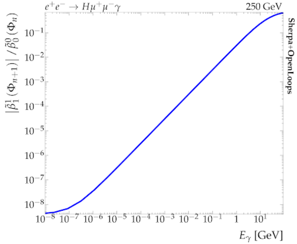
$$\tilde{\beta}_1^1\left(\Phi_{n+1}\right) = \frac{1}{2(2\pi)^3}\left|\mathcal{M}_0^{\frac{1}{2}}\left(\Phi_{n+1}\right)\right|^2 - \sum_{ij}\tilde{\mathcal{D}}_{ij}\left(\Phi_{ij+1}\otimes\Phi_n\right)$$

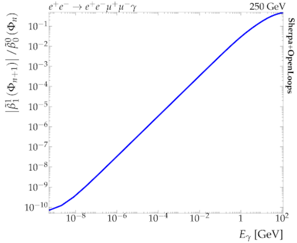
IR Finite contribution for real corrections

Real emission squared amplitude

YFS Subtraction term

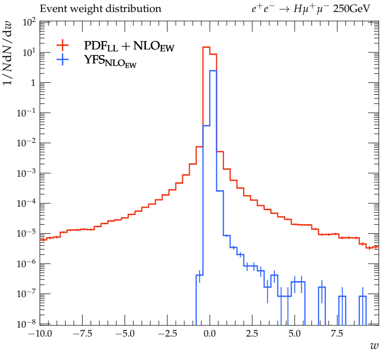
- Expansion of resummed soft real in YFS provides IR counterterms for exact real





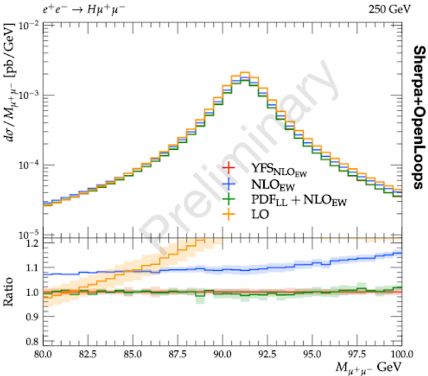
YFS Matching: Efficiency

- Tera-Z will require a significantly larger number of events to be generated than LHC
- Statistical power of event sample quickly becomes an issue
- YFS produces significantly smaller fraction of negative weighted events than e.g. CS subtraction



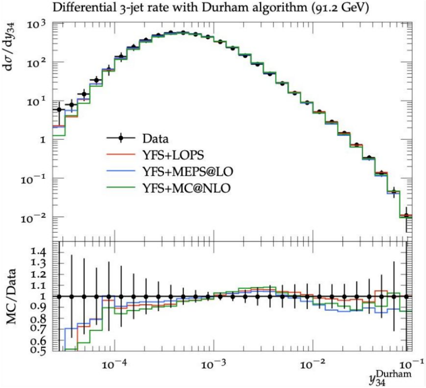
YFS Case Studies

- Higgs-Strahlung at FCC



A.Price, F.Krauss et al 25XY.abcd

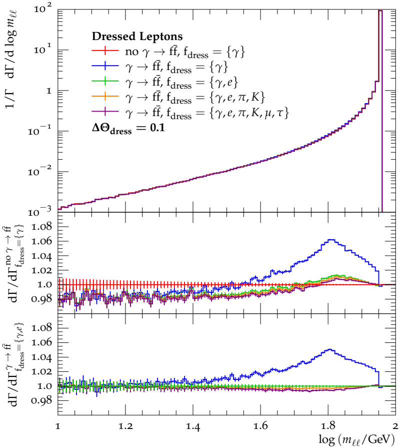
- QED ISR effects at the Z pole



Photon splittings

[Flower,Schönherr] arXiv:2210.07007

- YFS resummation does not include $\gamma \to f\bar{f}$ corrections, which enter at $\mathcal{O}(\alpha^2) \to$ must be added explicitly
- Evolve in parton-shower picture until photon virtuality drops below $m_{f\bar{f}}$
- Dress additional charges with additional photons using YFS



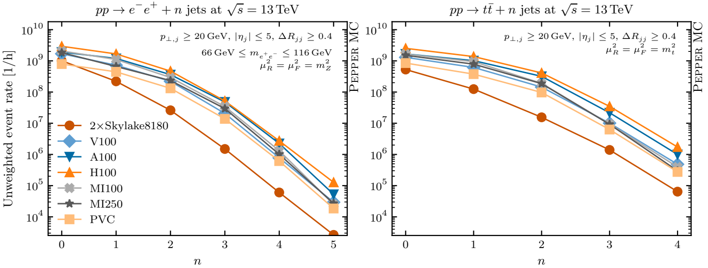
Efficient Computing

- Modern computing $\rightarrow$ many vendors & heterogeneous architectures
- (Pre-)Exascale computing systems intentionally diverse

Efficient Computing

[Bothmann et al.] arXiv:2311.06198

- Performance portability a major topic of discussion
  Driven by trends in computing industry and development at LCFs
- Portability of new codes achieved by Kokkos

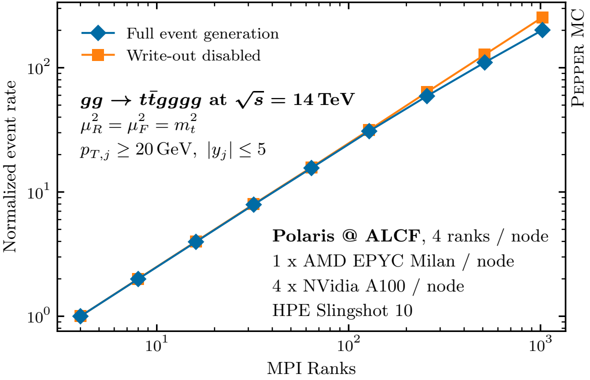


MEvents / hour	2×Skylake8180	V100	A100	H100	MI100	MI250	PVC
$pp \to t\bar{t} + 4j$	0.06	0.5	1.0	1.7	0.4	0.3	0.3
$pp \to e^- e^+ + 5j$	0.003	0.03	0.05	0.1	0.03	0.03	0.02

Efficient Computing

[Bothmann et al.] arXiv:2311.06198, arXiv:2309.13154

- Scalability achieved for up to 1024×A100's (1/2 of Polaris)
- Scaling violations only due to I/O, no issue for few nodes / low data volume
  Will have to be assessed once computing model for Tera-Z has taken shape



Summary & Discussion

- Perturbative QCD on track to deliver sufficient precision for FCC-ee
  Physics performance likely limited by understanding of hadronization
- Tera-Z will require highest MC statistics of any experiment so far
  May only be achievable with the help of HPC, possibly LCFS
- EWPOs will require multi-loop QED / EW calculations
  Must be implemented in MCs, at least partially
- Some of these developments overlap with LHC, some do not
  Mechanism needed for WFD and retention of talented developers

Backup Slides

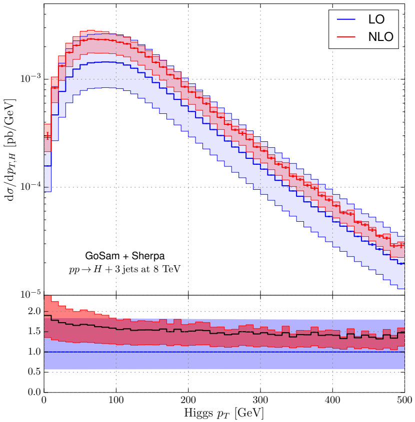
When fixed-order calculations don’t work ...

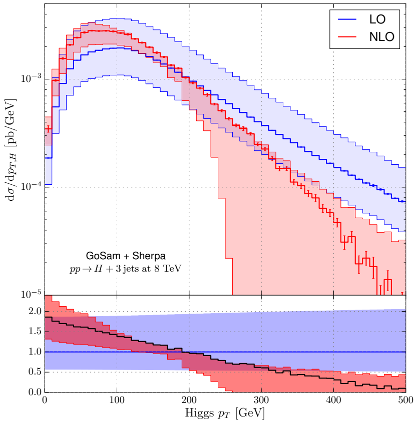
... one of the assumptions of fixed-order pQCD must be violated

- Scale hierarchies or rapidity gaps become large, leading to logarithmically enhanced corrections
- Flavor content of jets is resolved in some detail such that specifics of fragmentation are relevant
- Flavor channels have been down-selected to simplify computation (e.g. 4-flavor scheme in inclusive region)
- Scales are chosen inappropriately
- ...

Poor performance example: Scale choice

[Greiner et al.] arXiv:1506.01016



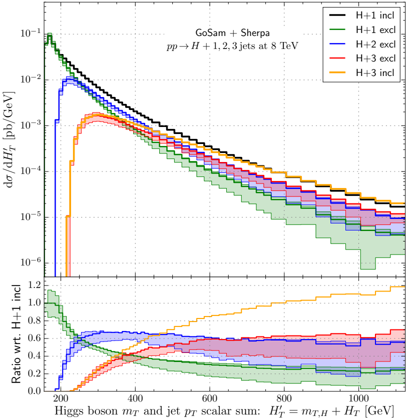


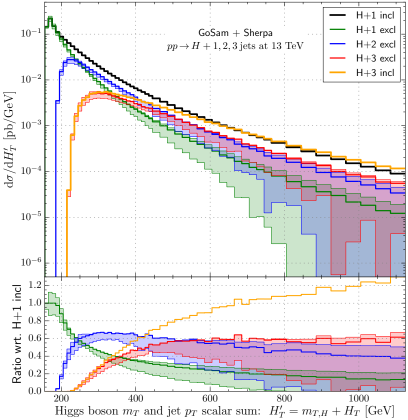
- H_T' – based scale
- Small uncertainties

- m_H – based scale
- Large uncertainties

Poor performance example: Jet cuts

[Greiner et al.] arXiv:1506.01016





- 8 TeV cms energy
- 13 TeV cms energy

Coherent soft vector boson radiation

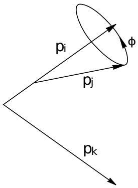
[Marchesini,Webber] NPB310(1988)461

- Soft vector boson emission can be understood in terms of squared current

$$J_\mu J^\mu \to \frac{p_i p_k}{(p_i p_j)(p_j p_k)} = \frac{W_{ik,j}}{E_j^2}$$

Angular "radiator" function

$$W_{ik,j} = \frac{1 - \cos\theta_{ik}}{(1 - \cos\theta_{ij})(1 - \cos\theta_{jk})}$$



- Divergent as $\theta_{ij} \to 0$ and as $\theta_{jk} \to 0$

$\to$ Expose individual collinear singularities using $W_{ik,j} = \tilde{W}^i_{ik,j} + \tilde{W}^k_{ki,j}$

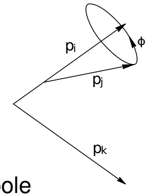
$$\tilde{W}^i_{ik,j} = \frac{1}{2}\left[\frac{1 - \cos\theta_{ik}}{(1 - \cos\theta_{ij})(1 - \cos\theta_{kj})} + \frac{1}{1 - \cos\theta_{ij}} - \frac{1}{1 - \cos\theta_{kj}}\right]$$

  - Divergent as $\theta_{ij} \to 0$, but regular as $\theta_{kj} \to 0$
  - Convenient properties upon integration over azimuthal angle

Coherent soft vector boson radiation

- Work in a frame where direction of $\vec{p}_i$ aligned with z-axis

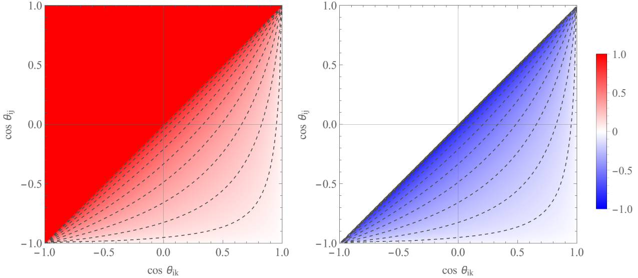
  $$\cos\theta_{kj} = \cos\theta_k^i \cos\theta_j^i + \sin\theta_k^i \sin\theta_j^i \cos\phi_{kj}^i$$



- Integration over ϕ_j yields

  $$\frac{1}{2\pi}\int_0^{2\pi} \mathrm{d}\phi_{kj}^i \tilde{W}_{ik,j}^i = \frac{1}{1-\cos\theta_j^i} \times \begin{cases} 1 & \text{if} \quad \theta_j^i < \theta_k^i \\ 0 & \text{else} \end{cases}$$

  - On average, no radiation outside cone defined by parent dipole
  - Differential radiation pattern more intricate:
    Positive & negative contributions outside cone sum to zero



Coherent soft vector boson radiation

[Marchesini,Webber] NPB330(1990)261

- Singularity in massive radiator screened by velocity $\rightarrow$ deadcone $\theta_0 \approx m/E$

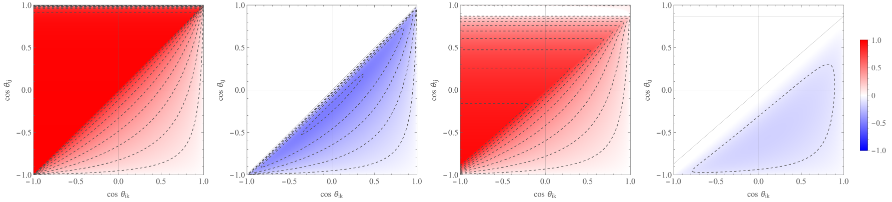
$$W_{ik,j} = \frac{1 - v_i v_k \cos\theta_{ik}}{(1 - v_i \cos\theta_{ij})(1 - v_k \cos\theta_{jk})} - \frac{(1 - v_i^2)/2}{(1 - v_i \cos\theta_{ij})^2} - \frac{(1 - v_k^2)/2}{(1 - v_k \cos\theta_{jk})^2}$$

- Quasi-collinear divergence if $m_Q \propto k_T$ as $k_T \rightarrow 0$
  $\rightarrow$ Expose individual singularities via $W_{ik,j} = \tilde{W}^i_{ik,j} + \tilde{W}^k_{ki,j}$

$$\tilde{W}^i_{ik,j} = \frac{1}{2(1 - v_i \cos\theta_{ij})}\left[\left(\frac{1 - v_i v_k \cos\theta_{ik}}{1 - v_k \cos\theta_{kj}} - \frac{1 - v_i^2}{1 - v_i \cos\theta_{ij}}\right) + 1 - \frac{1 - v_i \cos\theta_{ij}}{1 - v_k \cos\theta_{kj}}\right]$$

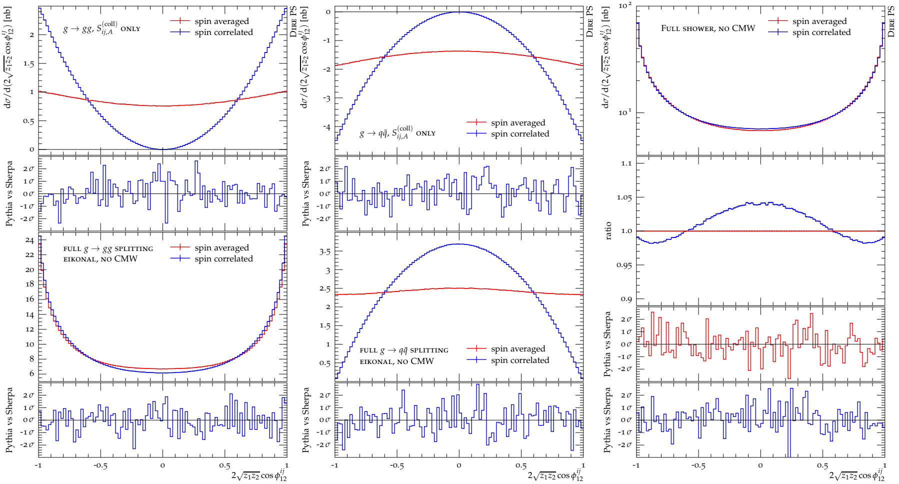
- Approximate angular ordering after azimuthal averaging

$v^2 = 1 - m_b^2/m_Z^2$ $\qquad\qquad$ $v^2 = 1 - m_t^2/(350\ \mathrm{GeV})^2$



Impact of spin correlations

[Dulat,Prestel,SH] arXiv:1805.03757



- Spin effects at $\mathcal{O}(\alpha_s^2)$ from double-soft / triple-collinear radiation pattern
- Is overall impact larger than QCD uncertainties?

MC@NLO matching

[Frixione,Webber] hep-ph/0204244

- Matched prediction given by MC@NLO master formula

$$\langle O\rangle = \int \mathrm{d}\Phi_B\, \bar{\mathrm{B}}^{(\mathrm{K})}(\Phi_B)\, \mathcal{F}^{(0)}_{\mathrm{MC}}(\mu_Q^2, O) + \int \mathrm{d}\Phi_R\, \mathrm{H}^{(\mathrm{K})}(\Phi_R)\, \mathcal{F}^{(1)}_{\mathrm{MC}}(t(\Phi_R), O)$$

- NLO-weighted Born cross section and hard remainder defined as

$$\bar{\mathrm{B}}^{(\mathrm{K})}(\Phi_B) = \mathrm{B}(\Phi_B) + \tilde{\mathrm{V}}(\Phi_B) + \mathrm{I}(\Phi_B) + \int \mathrm{d}\Phi_1 \Big[\mathrm{B}(\Phi_B)\,\mathrm{K}(\Phi_1) - \mathrm{S}(\Phi_R)\Big]$$

$$\mathrm{H}^{(\mathrm{K})}(\Phi_R) = \mathrm{R}(\Phi_R) - \mathrm{B}(\Phi_B)\,\mathrm{K}(\Phi_1)$$

- Parton shower described by generating functional $\mathcal{F}_{\mathrm{MC}}$

$$\langle O\rangle = \int \mathrm{d}\Phi_B\, \bar{\mathrm{B}}^{(\mathrm{K})}(\Phi_B)\, \mathcal{F}^{(0)}_{\mathrm{MC}}(\mu_Q^2, O) + \int \mathrm{d}\Phi_R\, \mathrm{H}^{(\mathrm{K})}(\Phi_R)\, \mathcal{F}^{(1)}_{\mathrm{MC}}(t(\Phi_R), O)$$

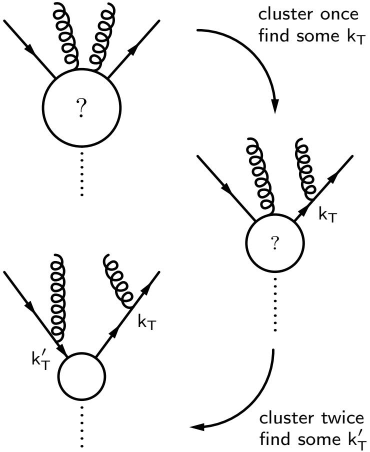
  Probability conservation: $\mathcal{F}_{\mathrm{MC}}(t,1) = 1 \rightarrow$ cross section correct at NLO

- Parametrically $\mathcal{O}(\alpha_s)$ correct, preserves logarithmic accuracy of PS

Leading order multi-jet merging

[André,Sjöstrand] hep-ph/9708390

- Start with a "core" process
  for example $e^+e^- \to q\bar{q}$
- This process is considered inclusive
  It sets the resummation scale μ_Q^2
- Higher-multiplicity ME can be
  reduced to core by clustering
  - Identify most likely splitting
    according to PS emission probability
  - Combine partons into mother
    according to PS kinematics
  - Continue until core process reached

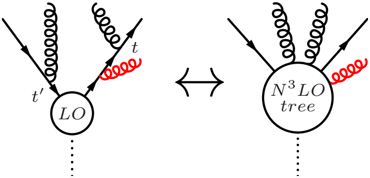


Leading order multi-jet merging

[Catani,Krauss,Kuhn,Webber] hep-ph/0109231, [Lönnblad] hep-ph/0112284

- Fixed-order calculation lacks resummed virtual corrections
- Most efficiently computed using pseudo-showers

- Start PS from core process
- Evolve until predefined branching
  ↔ truncated parton shower
- Emissions that would produce
  additional hard jets
  lead to event rejection (veto)



- Most efficient & accurate algorithm: CKKW-L [Lönnblad] hep-ph/0112284

YFS Matching: Real-Virtual Corrections

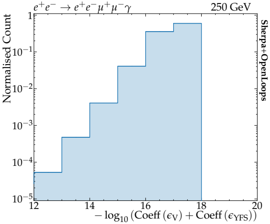
$$\tilde{\beta}_1^2\left(\Phi_{n+1}\right) = \mathscr{R}\mathscr{V}\left(\Phi_{n+1}\right) - \sum_{ij} \mathscr{D}_{ij}^{(1)}\left(\Phi_{ij+1} \otimes \Phi_n\right)$$

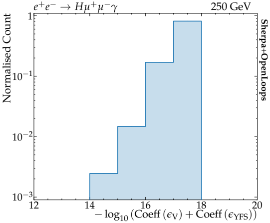
IR Finite one-loop contribution with additional real emission

Full One-loop amplitude

YFS Subtraction term

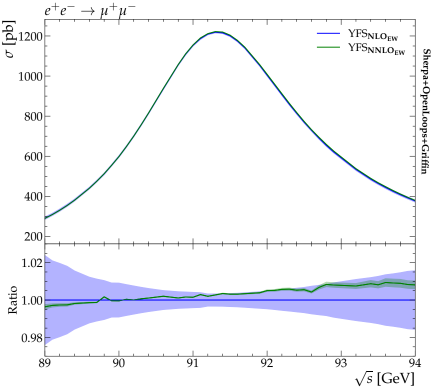
- Expansion of resummed soft real in YFS provides IR counterterms for exact real

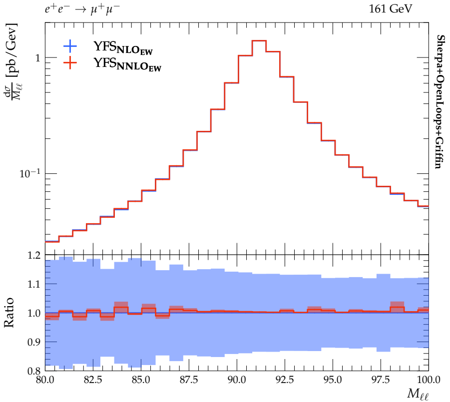




YFS @ NNLO

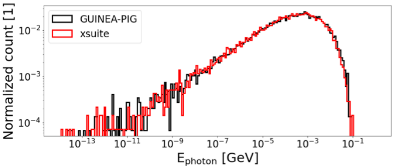
[A.Price] TBP





Beam-Beam Dynamics

- "Long distance" interaction between beam bunches $\rightarrow$ Beamstrahlung emission of photons
- Circular collider: Multiple beam passes combined with beam optics prevents the Beamstrahlung from accumulating $\rightarrow$ Gaussian distribution



[Kicsiny, Buffat, Iadarola, Pieloni, Schulte, Seidel]

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