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The Alaric Project

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LoopFest XXIV

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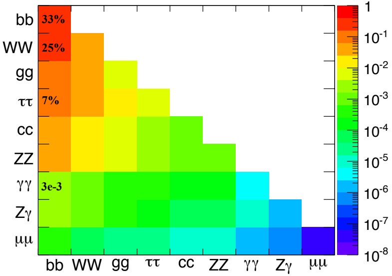
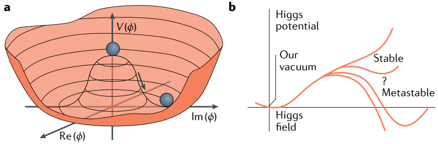
What we are preparing for

- Higgs self interaction is key to understanding of EW sector
- Measurement will require careful combination of many analyses with full HL-LHC data set
- Heavy flavor channels needed for high statistical significance



[J. Alison] LHCP ’24

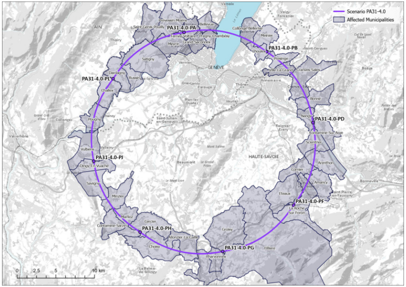


[Bass, DeRoeck, Kado] Nat. Rev. Phys. 3 (2021) 608

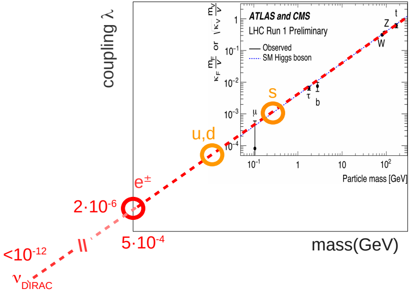
- Predictions for heavy quark production as part of inclusive heavy plus light flavor jets difficult to obtain at high precision
- Precise extraction of / limit setting on triple Higgs coupling depends crucially on understanding of all final states

What we are preparing for

- Unprecedented luminosity at Tera-Z option of a potential FCC-ee would leave no room for mis-modeling of perturbative and non-perturbative QCD effects



[CERN] https://home.cern/science/accelerators/



[D. d'Enterria] FCC week '24

- Extraction of Higgs Yukawa couplings would depend on precise modeling of light / heavy flavor jet production and flavor dynamics

Near-term focus of the Alaric project

- Parton shower at high theoretical precision
  - Increased logarithmic accuracy
  - Fully differential splittings at NLO
- Fixed-order matching and merging
  - MC@NLO & MEPS@NLO
  - NNLO matching
- Integration into Sherpa event generator
  - Matching, merging & fusing
  - Hadronization tunes

Matching and Merging

MC@NLO matching

[Frixione,Webber] hep-ph/0204244

- Matched prediction given by MC@NLO master formula

$$\langle O\rangle = \int \mathrm{d}\Phi_B\,\bar{\mathrm{B}}^{(\mathrm{K})}(\Phi_B)\,\mathcal{F}^{(0)}_{\mathrm{MC}}(\mu_Q^2,O) + \int \mathrm{d}\Phi_R\,\mathrm{H}^{(\mathrm{K})}(\Phi_R)\,\mathcal{F}^{(1)}_{\mathrm{MC}}(t(\Phi_R),O)$$

- NLO-weighted Born cross section and hard remainder defined as

$$\bar{\mathrm{B}}^{(\mathrm{K})}(\Phi_B) = \mathrm{B}(\Phi_B) + \tilde{\mathrm{V}}(\Phi_B) + \mathrm{I}(\Phi_B) + \int \mathrm{d}\Phi_1 \Big[\mathrm{B}(\Phi_B)\,\mathrm{K}(\Phi_1) - \mathrm{S}(\Phi_R)\Big]$$

$$\mathrm{H}^{(\mathrm{K})}(\Phi_R) = \mathrm{R}(\Phi_R) - \mathrm{B}(\Phi_B)\,\mathrm{K}(\Phi_1)$$

- Parton shower described by generating functional $\mathcal{F}_{\mathrm{MC}}$

$$\langle O\rangle = \int \mathrm{d}\Phi_B\,\bar{\mathrm{B}}^{(\mathrm{K})}(\Phi_B)\,\mathcal{F}^{(0)}_{\mathrm{MC}}(\mu_Q^2,O) + \int \mathrm{d}\Phi_R\,\mathrm{H}^{(\mathrm{K})}(\Phi_R)\,\mathcal{F}^{(1)}_{\mathrm{MC}}(t(\Phi_R),O)$$

  Probability conservation: $\mathcal{F}_{\mathrm{MC}}(t,1) = 1 \to$ cross section correct at NLO
- Parametrically $\mathcal{O}(\alpha_s)$ correct, preserves logarithmic accuracy of PS

$e^+e^- \rightarrow$hadrons

[Krauss,Meinzinger,Reichelt,SH] arXiv:2507.22837

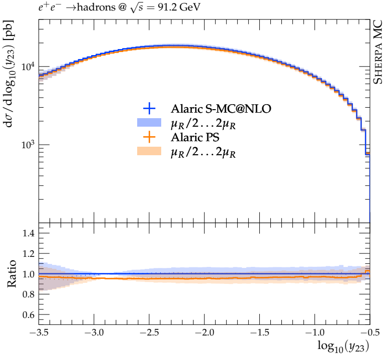
- Color- & spin-correct S-MC@NLO [Krauss,Schönherr,Siegert] arXiv:1111.1220
- All counterterms computed in analytic form

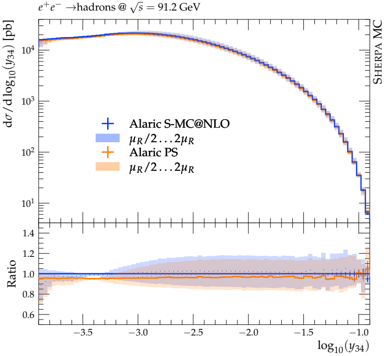
$\mu_R = \sqrt{s} = 91.2$GeV $y_{n,n-1} > (10\,\text{GeV})^2/s$		I [pb]	ΔI [pb]	RS [pb]	ΔRS [pb]	IRS [pb]	ΔIRS [pb]	ΔIRS/NLO
n=3	CS	5118.85	0.37	-1034.4	1.4	4084.45	1.4	0.1‰
	Alaric	5174.47	0.38	-1087.6	1.8	4086.84	1.8	0.1‰
	CS-Alaric					-2.39 ± 2.32		
n=4	CS	938.97	0.12	-94.45	0.73	844.52	0.74	0.5‰
	Alaric	922.36	0.12	-79.75	0.70	842.61	0.71	0.5‰
	CS-Alaric					1.91 ± 1.03		
n=5	CS	46.1544	0.015	-2.341	0.058	43.813	0.060	1‰
	Alaric	44.6125	0.014	-0.857	0.067	43.756	0.068	1‰
	CS-Alaric					0.057 ± 0.091		

- Fixed-order validation against Catani-Seymour subtraction

$e^+e^- \to$hadrons

[Krauss,Meinzinger,Reichelt,SH] arXiv:2507.22837

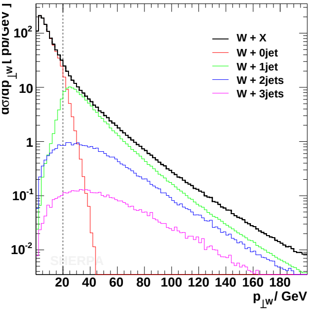




- Color- & spin-correct S-MC@NLO [Krauss,Schönherr,Siegert] arXiv:1111.1220
- All counterterms available in analytic form

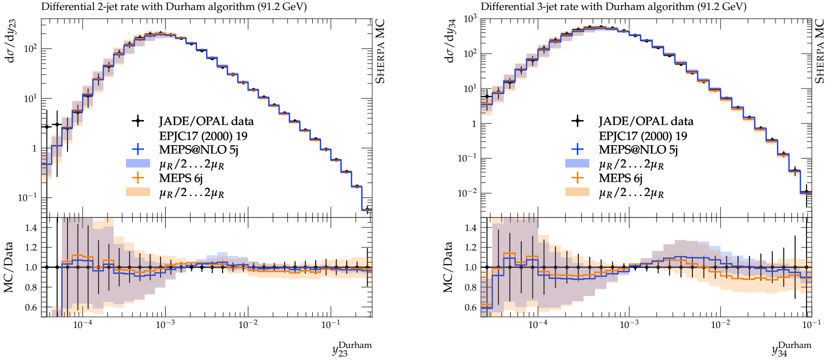
Next-to-leading order multi-jet merging

- Idea: Partition phase space into "hard" and "soft" region
- Parton shower populates soft domain
- NLO calculations replace PS emission in hard domain
- Need criterion to define "hard" & "soft" $\rightarrow$ jet measure Q (e.g. k_T jet resolution) and corresponding cut, Q_{cut}



$e^+e^- \to$ hadrons

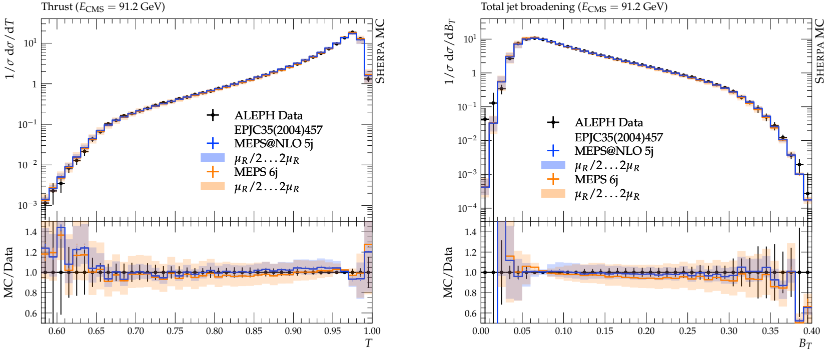
[Krauss,Meinzinger,Reichelt,SH] arXiv:2507.22837



- First merged prediction with up to 5 jets at NLO precision
- Radiation & splitting separated [Assi,SH] arXiv:2307.00728

$e^+e^- \to$hadrons

[Krauss,Meinzinger,Reichelt,SH] arXiv:2507.22837



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A new take on gluon polarization

Rethinking splitting functions

[Campbell,Knobbe,Preuss,Reichelt,SH] arXiv:2505.10408
[LeBlanc,Roloff,Whitman,SH] arXiv:2512.07025

- Gordon decomposition [Gordon] ZeitPhys140(1928)630

$$\frac{\not{p}+\not{q}}{(p+q)^2}\,T^a_{ij}\gamma^\mu = T^a_{ij}\left[\,S^\mu(p,q)+\frac{i\sigma^{\nu\mu}q_\nu}{(p+q)^2}-\frac{\gamma^\mu\not{p}}{(p+q)^2}\,\right]$$

  - Leading and sub-leading (LBK!) soft behavior given by scalar current [Gell-Mann,Goldberger] PR96(1954)1433, [Brown,Goble] PR173(1968)1505

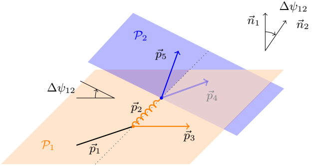
$$S^\mu(p,q)=\frac{(2p+q)^\mu}{(p+q)^2}$$

  - Magnetic term $\sigma^{\nu\mu}=i/2[\gamma^\nu,\gamma^\mu]$ due to quark spin
    $\gamma^\mu\not{p}$ generates seagull interactions of scalar theory
- Decomposition of triple & quartic gluon vertex even simpler
- Both decompositions hold at amplitude squared level [Chen et al.] arXiv:1404.5963
- **Scalar splitting functions & spin-dependent remainders**
  Clean identification of overlap beyond kinematical limits
- At 1-loop level, Background Field Method allows to derive
  Scalar radiators that satisfy the naive Ward identities
  $\rightarrow$ Extension of soft current [Catani,Grazzini] hep-ph/0007142

Application to gluon polarization

[Hoppe,Reichelt,SH] arXiv:2508.19018

- Conventional wisdom: Gluon spin correlations are a quantum effect



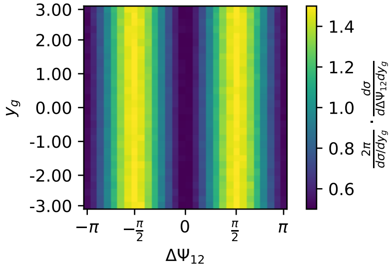
[Chen,Moult,Zhu] arXiv:2011.02492

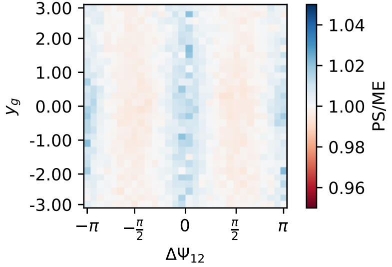
[Karlberg,Salam,Scyboz,Verheyen] arXiv:2103.16526

- Re-analyze using a new formalism → Most correlations are classical

[Staelin,Morgenthaler,Kong] *Electromagnetic Waves*, Pearson (1993)

- Reproduces known results, both in collinear & wide-angle region





Algorithm for polarized parton evolution

[Hoppe,Reichelt,SH] arXiv:2508.19018

- If gluon emitted off scalar dipole, store dipole polarization (i.e. current):

$$j_p^\mu(p_i,p_j;q) = \frac{J_{ij}^\mu(q)}{\sqrt{-J_{ij}^\nu(q)J_{ij,\nu}(q)}}\,, \quad \text{where} \quad J_{ij}^\mu(q) = S^\mu(p_i,q) - S^\mu(p_j,q)$$

- If gluon splits according to $k_\perp$-dependent part of $g \to gg$ or $g \to q\bar{q}$:
  - If polarization vector defined for this gluon:
    Contract with decay polarization $j_d^\mu(q_i,q_j)$
    Reweight splitting probability by $(j_p^\mu j_{d,\mu})^2$
    If $g \to gg$, mark new gluons as correlated
  - If polarization vector undefined for this gluon:
    Define polarization vector for correlated partner
    as polarization vector of the decay $j_d^\mu(q_i,q_j)$
- If gluon emits another gluon, transfer polarization
  and information on correlated partner to emitter

No spin density matrices. Not restricted to any kinematical limit.
Optimal scaling (linear in number of particles, both time & memory)

Classical effects

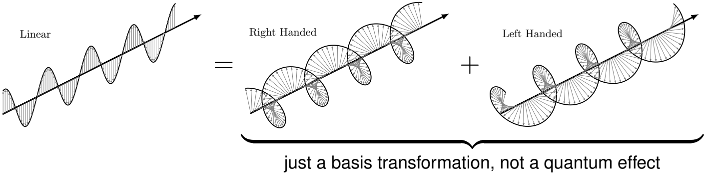
Recall your E&M lectures [Staelin,Morgenthaler,Kong] *Electromagnetic Waves*, Pearson (1993)

- Dipole antennae create linearly polarized EM waves
- Receivers must be co-polarized for maximum power transfer

QCD charge dipoles are *literally* dipole antennae

But what about conventional wisdom ...

- Do large off-diagonal entries in spin density matrices indicate that spin correlations are non-trivial quantum interferences?
- Not necessarily ... we could just be using an unsuitable basis:



Recall: **Dipoles only generate *linearly* polarized waves**

Quantum effects

- First actual quantum effect: Correlation of the newly produced gluons in the $k_\perp$-dependent part of a collinear $g \to gg$ splitting
  [Webber] Phys.Lett.B193(1987)91

  This is correctly captured by the new algorithm
  Still just a correlation, not an interference

- *Quantum interferences* require at least six external QCD charges
  $\to$ 3 production / decay planes defined by, say, K^μ, L^μ, M^μ

$$\frac{K_\rho K_\sigma L_\alpha L_\beta M^\mu M^\nu}{(\tilde{M}\tilde{L})(\tilde{M}\tilde{K})(\tilde{L}\tilde{K})}\,\tilde{P}^{\rho\sigma,\alpha\beta}_{g\to g,\mu\nu,(\mathrm{i})}(p_1,p_2) \approx$$

$$2C_A\,z_2\left[\frac{\cos\theta_{\tilde{K}\tilde{p}_{1,2}}\cos\theta_{\tilde{L}\tilde{p}_{1,2}}}{\cos\theta_{\tilde{K}\tilde{L}}}-\frac{\cos\theta_{\tilde{K}\tilde{p}_{1,2}}\cos\theta_{\tilde{M}\tilde{p}_{1,2}}}{\cos\theta_{\tilde{K}\tilde{M}}}\right]+\left(\begin{array}{c}K\leftrightarrow L\\ 1\leftrightarrow 2\end{array}\right)$$

  - Suppressed by z_2 or z_1
  - Azimuthal integral vanishes

  This is ***not captured*** **by the algorithm**
  But neither are other suppressed interferences, such as color monsters
  [Dokshitzer,Khoze,Mueller,Troyan] Basics of perturbative QCD, Editions Frontières (1991), Sec.6.3.3

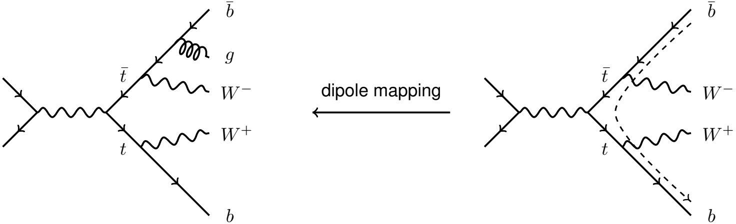
Resonance- and width-aware parton evolution

The Problem – Part I: Kinematics

- Alaric built on Catani-Seymour like dipole subtraction terms

$$\mathcal{D}_{ij,k} = -\frac{1}{2p_ip_j}\,{}_m\langle\tilde{1},\ldots,\widetilde{ij},\ldots,\widetilde{m+1}|\frac{\mathbf{T}_i\mathbf{T}_k}{\mathbf{T}_i^2}V_{ij,k}|\tilde{1},\ldots,\widetilde{ij},\ldots,\widetilde{m+1}\rangle_m$$

- Dipole momentum mapping affects top-quark virtuality



Induces multiplicative correction to Born cross section of the form

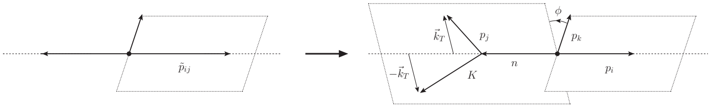
$$\frac{\left[(p_{W^+}+\tilde{p}_b)^2-m_t^2\right]^2+m_t^2\Gamma_t^2}{\left[(p_{W^+}+p_b+p_g)^2-m_t^2\right]^2+m_t^2\Gamma_t^2}\,\frac{\left[(p_{W^-}+\tilde{p}_{\bar{b}})^2-m_t^2\right]^2+m_t^2\Gamma_t^2}{\left[(p_{W^-}+p_{\bar{b}})^2-m_t^2\right]^2+m_t^2\Gamma_t^2}$$

$\rightarrow$ can be large, even if recoil effects of $\mathcal{O}(\Gamma_t)$

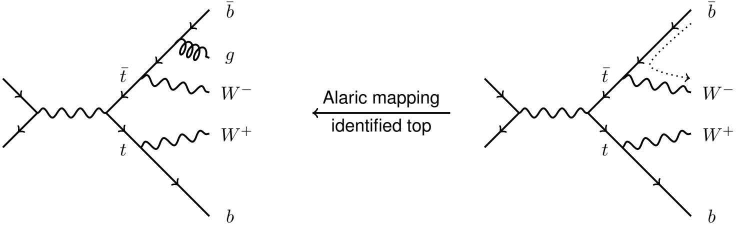
Resonance aware momentum mapping

[Liebschner,Siegert,SH] arXiv:1807.04348, [Reichelt,SH] arXiv:2604.13978

- Recoil vector in Alaric kinematics mapping can be chosen freely



- Choose to distribute recoil only on top-quark decay products



Top-quark virtualities unaffected $\rightarrow$ no changes to production rate

Resonance aware splitting functions

[Reichelt,SH] arXiv:2604.13978

- Alaric standard radiator given by

$$V_{ig,k}^{(s)}(p_i, p_j, n) = 8\pi\mu^{2\varepsilon}\alpha_s\, C_i\, \frac{2(p_ip_k)(p_in)}{(p_ip_j)(p_kn) + (p_kp_j)(p_in)}$$

  n – gauge vector connected to momentum mapping

- Recoil now distributed to W-bosons
  $\rightarrow$ spoils partial fractioning of eikonal
- Use standard Catani-Seymour technique
  for identified partons [Catani,Seymour] hep-ph/9605323

$$V_{ig,k}^{(s,\text{CS})}(p_i, p_j, n) = 8\pi\mu^{2\varepsilon}\alpha_s\, C_i\, \frac{2(p_ip_k)}{(p_ip_j) + (p_kp_j)}$$

  Requires re-computation of S-MC@NLO matching terms

The problem – Part II: Coherence effects

[Jikia] Phys.Lett.B257(1991)196, [Khoze,Stirling,Orr] Nucl.Phys.B378(1992)413

- ME squared above & close to threshold consists mainly of tb dipoles

$$B_i^2 = \frac{2p_iq_i}{(p_ip_j)(p_jq_i)} - \frac{m_t^2}{(p_jq_i)^2}\,,$$

p_i – b momentum
q_i – t momentum
p_j – g momentum

- Additional contributions from the $t\bar{t}$ dipole suppressed by top-quark velocity
- But width suppressed interference survives, even close to threshold:

$$2\text{Re}[B_iB_k^*] = \frac{\Gamma_t^2}{E_j^2+\Gamma_t^2}\left(\frac{q_iq_k}{(q_ip_j)(p_jq_k)} - \frac{p_iq_k}{(p_ip_j)(p_jq_k)} + \frac{p_ip_k}{(p_ip_j)(p_kp_j)} - \frac{p_kq_i}{(p_kp_j)(p_jq_i)}\right)$$

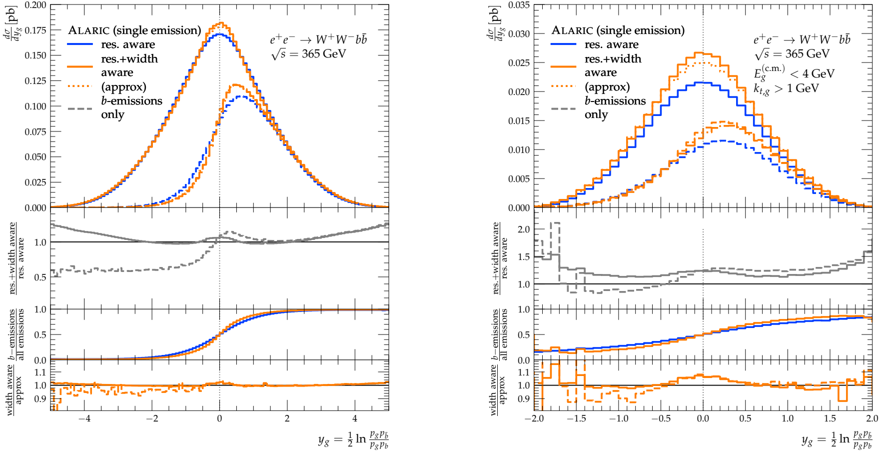
- Defines width-aware splitting kernel

$$V_{ij,k}^{(s,\text{w})}(p_i,p_j,p_k,q_i) = V_{ij,k}^{(s,\text{CS})}(p_i,p_j,p_k) + 8\pi\mu^{2\varepsilon}\alpha_s\,C_i\,\chi(z)\left(\frac{2(p_iq_i)}{(p_ip_j)+(q_ip_j)} - \frac{2(p_ip_k)}{(p_ip_j)+(p_kp_j)}\right)$$

where $\chi(z) = (1-z)^2/\big[(1-z)^2+\gamma^2\big]$

Width aware evolution

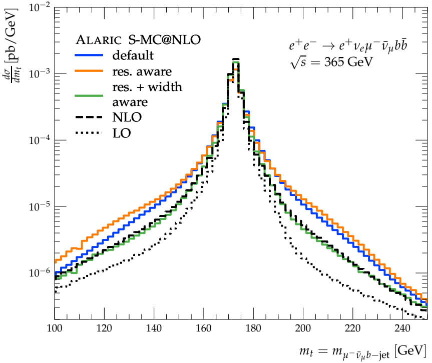
[Reichelt,SH] arXiv:2604.13978

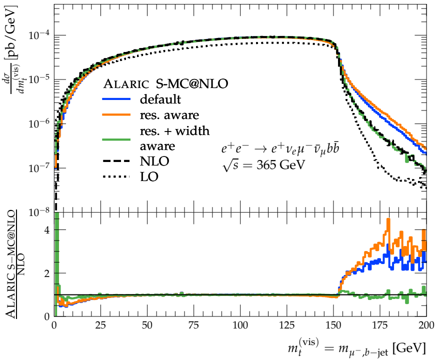


- Lund-plane rapidity of first parton-shower emission
- Width-awareness affects central region → emissions more forward

Width aware NLO matching

[Reichelt,SH] arXiv:2604.13978





- Truth-level reconstructed top-quark virtuality
- Particle-level reconstructed lepton-b-jet virtuality
- Standard vs. width-aware S-MC@NLO

Summary & Outlook

Current and future developments of Alaric:

- Higher-order corrections
  - NLO splitting functions
- Fixed-order matching
  - MC@NLO for initial-state evolution
  - Fully differential NNLO matching
- Practicalities
  - Release as part of Sherpa 3.1.x
  - Several components available in Python