FERMILAB-SLIDES-25-139-T

The Alaric Project

Stefan Höche

Fermi National Accelerator Laboratory

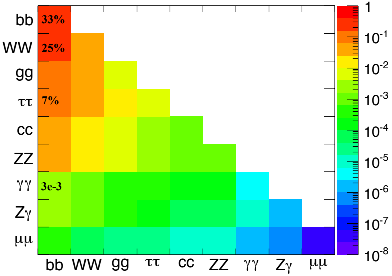
Parton Showers and Resummation

CERN, 07/15/2025

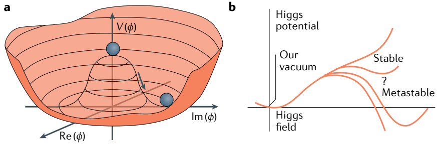
This manuscript has been authored by FermiForward Discovery Group, LLC under Contract No. 89243024CSC000002 with the U.S. Department of Energy, Office of Science, Office of High Energy Physics.

What we are preparing for

- Higgs self interaction is key to understanding of EW sector
- Measurement will require careful combination of many analyses with full HL-LHC data set
- Heavy flavor channels needed for high statistical significance



[J. Alison] LHCP ’24

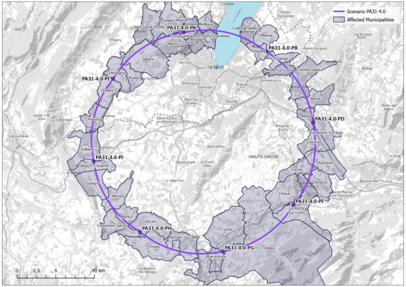


[Bass, DeRoeck, Kado] Nat. Rev. Phys. 3 (2021) 608

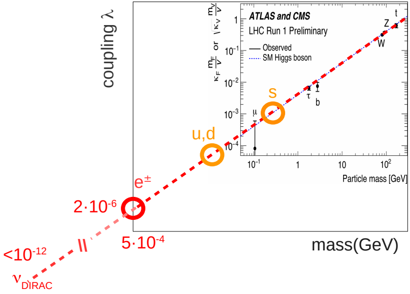
- Predictions for heavy quark production as part of inclusive heavy plus light flavor jets difficult to obtain at high precision
- Precise extraction of / limit setting on triple Higgs coupling depends crucially on understanding of all final states

What we are preparing for

- Unprecedented luminosity at Tera-Z option of a potential FCC-ee would leave no room for mis-modeling of non-perturbative QCD effects



[CERN] https://home.cern/science/accelerators/



[D. d'Enterria] FCC week '24

- Extraction of Higgs Yukawa couplings would depend on precise modeling of light / heavy flavor jet production and flavor dynamics

Near-term focus of the Alaric project

- Parton shower at high theoretical precision
  - Increased logarithmic accuracy
  - Fully differential splittings at NLO
- Fixed-order matching and merging
  - Automatic MC@NLO at fixed jet multiplicity
  - MEPS@NLO for combination of multiplicities
- Integration into Sherpa event generator
  - Matching, merging & fusing for heavy quarks
  - Hadronization tunes

Evolution with massless quarks

Additive soft-collinear matching

[Marchesini,Webber] NPB310(1988)461

- Soft gluon radiator can be written in terms of energies and angles

$$J_\mu J^\mu \to \frac{p_i p_k}{(p_i p_j)(p_j p_k)} = \frac{W_{ik,j}}{E_j^2}$$

Angular "radiator" function

$$W_{ik,j} = \frac{1-\cos\theta_{ik}}{(1-\cos\theta_{ij})(1-\cos\theta_{jk})}$$

- Divergent as $\theta_{ij} \to 0$ and as $\theta_{jk} \to 0$

$\to$ Expose individual collinear singularities using $W_{ik,j} = \tilde{W}^i_{ik,j} + \tilde{W}^k_{ki,j}$

$$\tilde{W}^i_{ik,j} = \frac{1}{2}\left[\frac{1-\cos\theta_{ik}}{(1-\cos\theta_{ij})(1-\cos\theta_{kj})} + \frac{1}{1-\cos\theta_{ij}} - \frac{1}{1-\cos\theta_{kj}}\right]$$

  - Divergent as $\theta_{ij} \to 0$, but regular as $\theta_{kj} \to 0$
  - Convenient properties upon integration over azimuthal angle

Additive soft-collinear matching

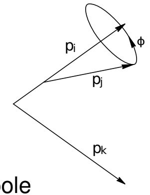
- Work in a frame where direction of $\vec{p}_i$ aligned with z-axis

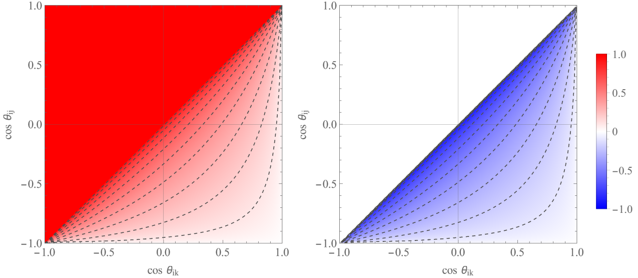
  $$\cos\theta_{kj} = \cos\theta_k^i \cos\theta_j^i + \sin\theta_k^i \sin\theta_j^i \cos\phi_{kj}^i$$

- Integration over ϕ_j yields

  $$\frac{1}{2\pi}\int_0^{2\pi} \mathrm{d}\phi_{kj}^i \tilde{W}_{ik,j}^i = \frac{1}{1-\cos\theta_j^i} \times \begin{cases} 1 & \text{if} \quad \theta_j^i < \theta_k^i \\ 0 & \text{else} \end{cases}$$

  - On average, no radiation outside cone defined by parent dipole
  - Differential radiation pattern more intricate:
    Positive & negative contributions outside cone sum to zero



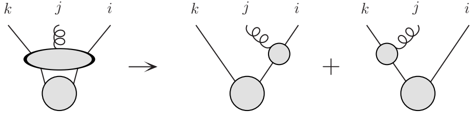


Multiplicative soft-collinear matching

[Herren,Krauss,Reichelt,Schönherr,SH] arXiv:2208.06057

- Alternative to additive matching: partial fraction matrix element & match to collinear sectors [Ellis,Ross,Terrano] NPB178(1981)421, [Catani,Seymour] hep-ph/9605323

$$\frac{W_{ik,j}}{E_j^2} \to \frac{1}{p_i p_j}\frac{p_i p_k}{(p_i+p_k)p_j} + \frac{1}{p_k p_j}\frac{p_i p_k}{(p_i+p_k)p_j}$$



  - Captures matrix element both in angular ordered and unordered region
  - Caveat: Oversampling difficult for certain kinematics maps
- Separate into energy & angle first
  Partial fraction angular radiator only: $W_{ik,j} = \bar{W}^i_{ik,j} + \bar{W}^k_{ki,j}$

$$\bar{W}^i_{ik,j} = \frac{1-\cos\theta_{ik}}{(1-\cos\theta_{ij})(2-\cos\theta_{ij}-\cos\theta_{kj})}$$

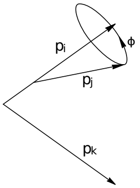
  - Bounded by $(1-\cos\theta_{ij})\bar{W}^i_{ik,j} < 2$
  - Strictly positive

Multiplicative soft-collinear matching

- Integration over ϕ_j yields

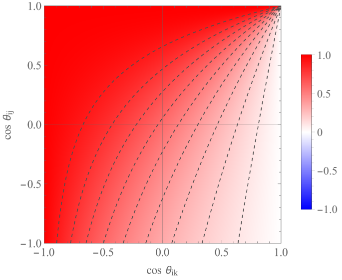
$$\frac{1}{2\pi}\int_0^{2\pi} \mathrm{d}\phi_{kj}^{i}\,\bar{W}_{ik,j}^{i} = \frac{1}{1-\cos\theta_j^i}\,\frac{1}{\sqrt{(\bar{A}_{ij,k}^{i})^2-(\bar{B}_{ij,k}^{i})^2}}$$

  - Radiation across all of phase space
  - Probabilistic radiation pattern

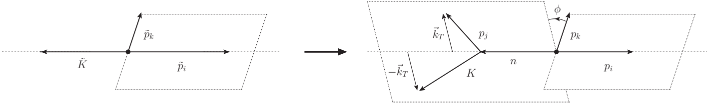


$$\bar{A}_{ij,k}^{i} = \frac{2-\cos\theta_j^i(1+\cos\theta_k^i)}{1-\cos\theta_k^i}$$

$$\bar{B}_{ij,k}^{i} = \frac{\sqrt{(1-\cos^2\theta_j^i)(1-\cos^2\theta_k^i)}}{1-\cos\theta_k^i}$$



Kinematics mapping



- In collinear limit, splitting kinematics defined by ($n \to$ auxiliary vector)

$$p_i \xrightarrow{i||j} z\,\tilde{p}_i\ , \qquad p_j \xrightarrow{i||j} (1-z)\,\tilde{p}_i \qquad \text{where} \qquad z = \frac{p_i n}{(p_i + p_j)n}$$

- Parametrization, using hard momentum $\tilde{K}$

$$p_i = z\,\tilde{p}_i\ , \qquad n = \tilde{K} + (1-z)\,\tilde{p}_i$$

- Using on-shell conditions & momentum conservation ($\kappa = \tilde{K}^2/(2\tilde{p}_i\tilde{K})$)

$$\begin{aligned} p_j &= (1-z)\,\tilde{p}_i + v\big(\tilde{K} - (1-z+2\kappa)\,\tilde{p}_i\big) + k_\perp \\ K &= \qquad\quad \tilde{K} - v\big(\tilde{K} - (1-z+2\kappa)\,\tilde{p}_i\big) - k_\perp \end{aligned}$$

- Momenta in $\tilde{K}$ Lorentz-boosted to new frame K [Catani,Seymour] hep-ph/9605323

$$p_l^\mu \to \Lambda^\mu{}_\nu(K,\tilde{K})\,p_l^\nu\ , \qquad \Lambda^\mu{}_\nu(K,\tilde{K}) = g^\mu{}_\nu - \frac{2(K+\tilde{K})^\mu(K+\tilde{K})_\nu}{(K+\tilde{K})^2} + \frac{2\tilde{K}^\mu K_\nu}{K^2}\ .$$

Recoil safety – Analytic proof

- Logarithmic accuracy of parton shower can be quantified by comparing results to (semi-)analytic resummation e.g. [Banfi,Salam,Zanderighi] hep-ph/0407286
- Example: Thrust or FC_0 in $e^+e^- \to$hadrons

- Define a shower evolution variable $\xi = k_T^2/(1-z)$
- Parton-shower one-emission probability for $\xi > Q^2\tau$

$$R_{\rm PS}(\tau) = 2\int_{Q^2\tau}^{Q^2}\frac{d\xi}{\xi}\int_{z_{\rm min}}^{z_{\rm max}} dz\,\frac{\alpha_s\left(k_T^2\right)}{2\pi}\,C_F\left[\frac{2}{1-z}-(1+z)\right]\Theta(\eta)$$

- Approximate to NLL accuracy

$$R_{\rm NLL}(\tau) = 2\int_{Q^2\tau}^{Q^2}\frac{d\xi}{\xi}\left[\int_0^1 dz\,\frac{\alpha_s\left(k_T^2\right)}{2\pi}\,\frac{2\,C_F}{1-z}\Theta(\eta)-\frac{\alpha_s(\xi)}{\pi}\,C_F B_q\right]$$

Recoil safety – Analytic proof

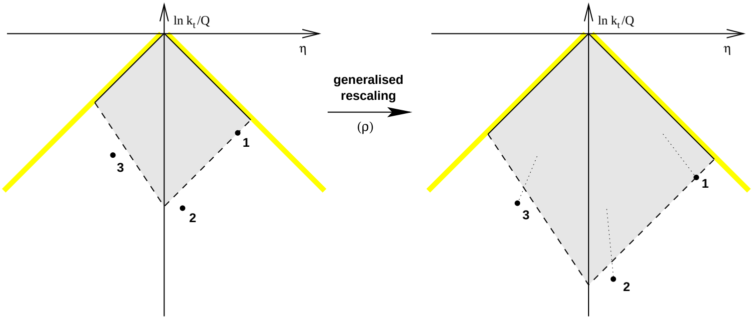
- Cumulative cross section $\Sigma(\tau) = e^{-R(\tau)}\mathcal{F}(\tau)$ obtained from all-orders resummed result by Taylor expansion of virtual corrections in cutoff ε

$$\mathcal{F}(\tau) = \int \mathrm{d}^3k_1 |M(k_1)|^2 \, e^{-R' \ln \frac{\tau}{\varepsilon v_1}} \sum_{m=0}^{\infty} \frac{1}{m!} \left(\prod_{i=2}^{m+1} \int_{\varepsilon v_1}^{v_1} \mathrm{d}^3k_i |M(k_i)|^2 \right) \times \Theta\big(\tau - V(\{p\}, k_1, \ldots, k_n)\big)$$

  - $\mathcal{F}(\tau)$ is pure NLL & accounts for (correlated) multiple-emission effects
- In order to make $\mathcal{F}(\tau)$ calculable, make the following assumptions
  - Observable is recursively infrared and collinear safe
  - Hold $\alpha_s(Q^2)\ln\tau$ fixed, while taking limit $\tau \to 0$
    $\to$ Can factorize integrals and neglect kinematic edge effects

    Can be interpreted as $\alpha_s \to 0$ or $s \to \infty$ limit

Recoil safety – Analytic proof



- $\alpha_s \to 0$ / $s \to \infty$ limit taken by similarity transformation of Lund plane
- Can be parametrized in terms of scaling parameter ρ

$$k_{t,l} \to k'_{t,l} = k_{t,l}\,\rho^{(1-\xi_l)/a+\xi_l/(a+b)}$$

$$\eta_l \to \eta'_l = \eta - \xi_l \frac{\ln \rho}{a+b}\,, \qquad \text{where} \qquad \xi = \frac{\eta}{\eta_{\max}}$$

observable parametrization at one-emission level: $v = (k_t^2/Q^2)^a \exp(-b\eta)$

- NLL precision requires scaling to be maintained after additional emissions

Recoil safety – Analytic proof

- Lorentz transformation defined by shift $\tilde{K} \to K$

$$K^\mu = \tilde{K}^\mu - X^\mu\;, \qquad \text{where} \qquad X^\mu = p_j^\mu - (1-z)\,\tilde{p}_i^\mu$$

- X is small, but is it small enough? Rewrite

$$\Lambda^\mu_{\ \nu}(K,\tilde{K}) = g^\mu_\nu + \tilde{K}^\mu A_\nu + X^\mu B_\nu$$

- In NLL limit, coefficients scale as

$$A^\nu \overset{\rho\to 0}{\longrightarrow} 2\,\frac{\tilde{K}X}{\tilde{K}^2}\,\frac{\tilde{K}^\nu}{\tilde{K}^2} - \frac{X^\nu}{\tilde{K}^2}\;, \qquad \text{and} \qquad B^\nu \overset{\rho\to 0}{\longrightarrow} \frac{\tilde{K}^\nu}{\tilde{K}^2}\;.$$

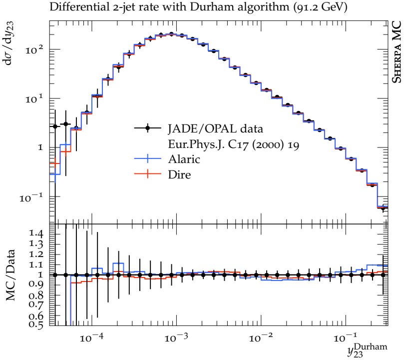
- Simplify situation by taking $a=1$, $b=0$ (worst offenders)
  Relative momentum shift of soft emission particle l becomes

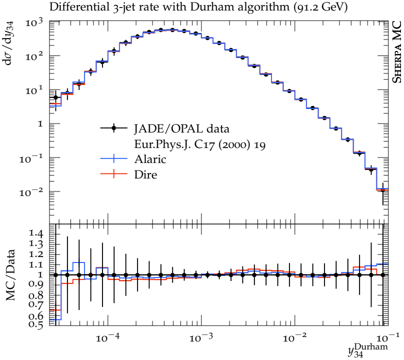
$$\Delta p_l^{0,3}/\tilde{p}_l^{0,3} \sim \rho^{1-\max(\xi_i,\xi_j)} \qquad \overset{\rho\to 0}{\longrightarrow} \quad 0$$
$$\Delta p_l^{1,2}/\tilde{p}_l^{1,2} \sim \rho^{1-\xi_l} \qquad \overset{\rho\to 0}{\longrightarrow} \quad 0$$

- For hard momenta, leading terms in X^μ cancel exactly
  Remaining components scale as ρ or stronger

$e^+e^- \rightarrow$ hadrons

[Herren,Krauss,Reichelt,Schönherr,SH] arXiv:2208.06057

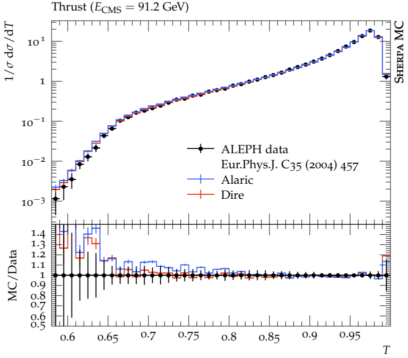


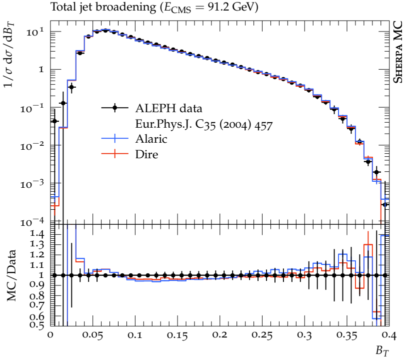


- Comparison to experimental data from LEP
- Radiation & splitting treated on same footing

$e^+e^- \to$ hadrons

[Herren,Krauss,Reichelt,Schönherr,SH] arXiv:2208.06057





- Comparison to experimental data from LEP
- Radiation & splitting treated on same footing

Evolution with massive quarks

Additive soft-collinear matching

[Marchesini,Webber] NPB330(1990)261

- Singularity in angular radiator screened by velocity → deadcone $\theta_0 \approx m/E$

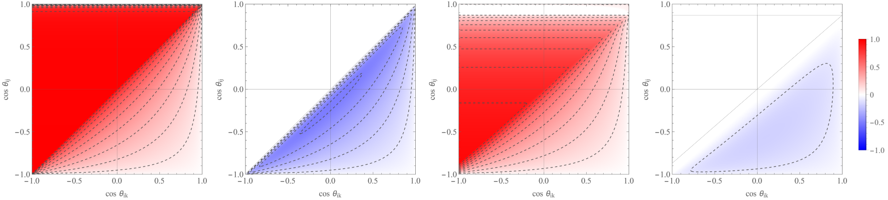
$$W_{ik,j} = \frac{1 - v_i v_k \cos\theta_{ik}}{(1 - v_i \cos\theta_{ij})(1 - v_k \cos\theta_{jk})} - \frac{(1 - v_i^2)/2}{(1 - v_i \cos\theta_{ij})^2} - \frac{(1 - v_k^2)/2}{(1 - v_k \cos\theta_{jk})^2}$$

- Quasi-collinear divergence if $m_Q \propto k_T$ as $k_T \to 0$
  → Expose individual singularities via $W_{ik,j} = \tilde{W}^i_{ik,j} + \tilde{W}^k_{ki,j}$

$$\tilde{W}^i_{ik,j} = \frac{1}{2(1 - v_i \cos\theta_{ij})}\left[\left(\frac{1 - v_i v_k \cos\theta_{ik}}{1 - v_k \cos\theta_{kj}} - \frac{1 - v_i^2}{1 - v_i \cos\theta_{ij}}\right) + 1 - \frac{1 - v_i \cos\theta_{ij}}{1 - v_k \cos\theta_{kj}}\right]$$

- Approximate angular ordering after azimuthal averaging

$v^2 = 1 - m_b^2/m_Z^2$ $\qquad$ $v^2 = 1 - m_t^2/(350\ \text{GeV})^2$



Multiplicative soft-collinear matching

[Assi,SH] arXiv:2307.00728

- Alternative approach: separate into energy & angle first
  Partial fraction angular radiator only: $W_{ik,j} = \bar{W}^i_{ik,j} + \bar{W}^k_{ki,j}$

$$\bar{W}^i_{ik,j} = \frac{1 - v_k \cos\theta_{kj}}{2 - v_i \cos\theta_{ij} - v_k \cos\theta_{kj}} \, W_{ik,j}$$

- Can be written in more intuitive form (n^μ defines reference frame)

$$\bar{W}^i_{ik,j} = \frac{1}{2 l_i l_j} \left(\frac{l^2_{ik}}{l_{ik} l_j} - \frac{l^2_i}{l_i l_j} - \frac{l^2_k}{l_k l_j} \right) , \qquad \text{where} \qquad l^\mu_i = \sqrt{n^2}\, \frac{p^\mu_i}{p_i n}$$

- Quasi-collinear limit manifest

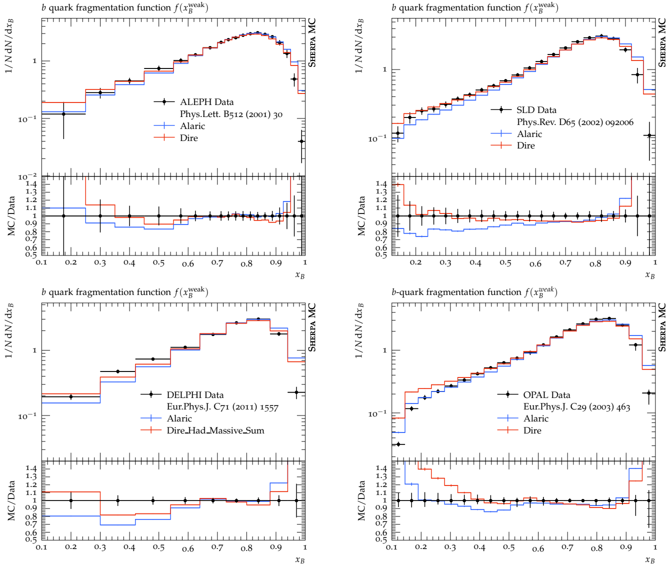
$$\frac{\bar{W}_{ik,j}}{E^2_j} \xrightarrow[m_i \propto p_i p_j]{i||j} w^{(\mathrm{coll})}_{ik,j}(z) := \frac{1}{2 p_i p_j} \left(\frac{2z}{1-z} - \frac{m^2_i}{p_i p_j} \right)$$

- Matching to massive DGLAP splitting functions

$$\frac{P_{(ij)i}(z,\varepsilon)}{(p_i + p_j)^2 - m^2_{ij}} \to \frac{P_{(ij)i}(z,\varepsilon)}{(p_i + p_j)^2 - m^2_{ij}} + \delta_{(ij)i}\, \mathbf{T}^2_i \left[\frac{\bar{W}^i_{ik,j}}{E^2_j} - w^{(\mathrm{coll})}_{ik,j}(z) \right] ,$$

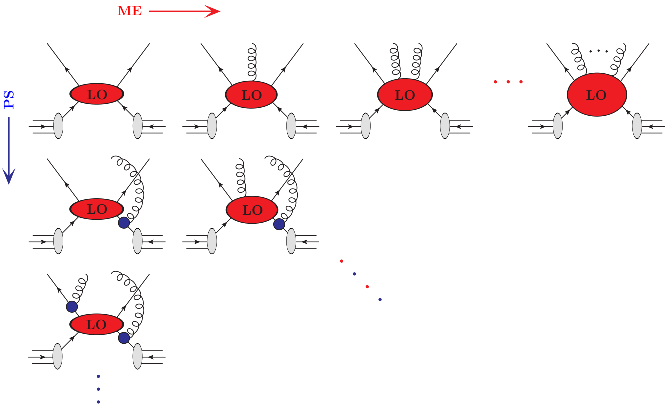
$e^+e^- \rightarrow$hadrons

[Assi,SH] arXiv:2307.00728



Matching and Merging

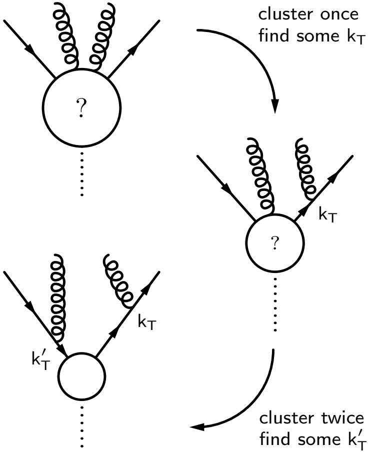
Leading order multi-jet merging



Leading order multi-jet merging

[André,Sjöstrand] hep-ph/9708390

- Start with a "core" process
  for example $e^+e^- \to q\bar{q}$
- This process is considered inclusive
  It sets the resummation scale μ_Q^2
- Higher-multiplicity ME can be reduced to core by clustering
  - Identify most likely splitting according to PS emission probability
  - Combine partons into mother according to PS kinematics
  - Continue until core process reached

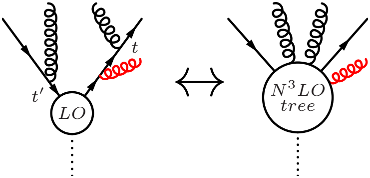


Leading order multi-jet merging

[Catani,Krauss,Kuhn,Webber] hep-ph/0109231, [Lönnblad] hep-ph/0112284

- Fixed-order calculation lacks resummed virtual corrections
- Most efficiently computed using pseudo-showers

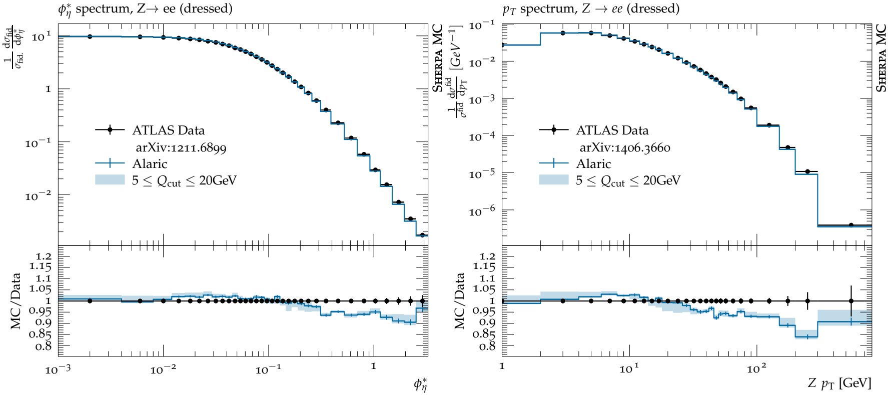
- Start PS from core process
- Evolve until predefined branching
  $\leftrightarrow$ truncated parton shower
- Emissions that would produce
  additional hard jets
  lead to event rejection (veto)



- Truncated unvetoed parton shower is ill-defined ($\nearrow$ PSR school)
- Alaric uses CKKW-L solution [Lönnblad] hep-ph/0112284

Drell-Yan lepton pair production

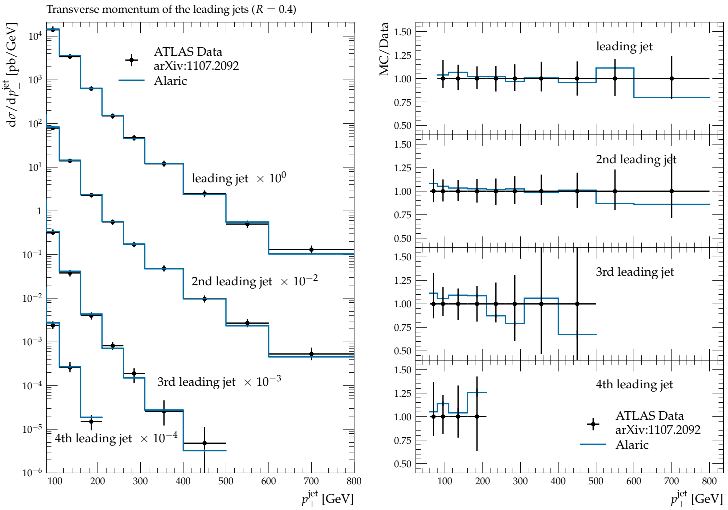
[Krauss,Reichelt,SH] arXiv:2404.14360



- Comparison to experimental data from LHC
- Leading-order multi-jet merging with up to two jets

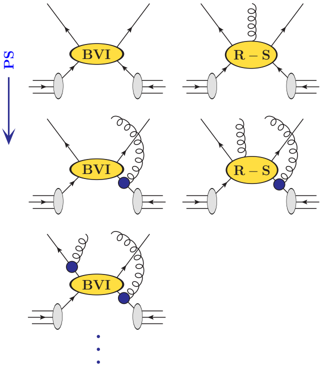
Jet production

[Krauss,Reichelt,SH] arXiv:2404.14360



- Comparison to experimental data from LHC, parton shower only

MC@NLO matching



MC@NLO matching

[Frixione,Webber] hep-ph/0204244

- Matched prediction given by MC@NLO master formula

$$\langle O\rangle = \int \mathrm{d}\Phi_B\,\bar{\mathrm{B}}^{(\mathrm{K})}(\Phi_B)\,\mathcal{F}^{(0)}_{\mathrm{MC}}(\mu_Q^2,O) + \int \mathrm{d}\Phi_R\,\mathrm{H}^{(\mathrm{K})}(\Phi_R)\,\mathcal{F}^{(1)}_{\mathrm{MC}}(t(\Phi_R),O)$$

- NLO-weighted Born cross section and hard remainder defined as

$$\bar{\mathrm{B}}^{(\mathrm{K})}(\Phi_B) = \mathrm{B}(\Phi_B) + \tilde{\mathrm{V}}(\Phi_B) + \mathrm{I}(\Phi_B) + \int \mathrm{d}\Phi_1 \Big[\mathrm{B}(\Phi_B)\,\mathrm{K}(\Phi_1) - \mathrm{S}(\Phi_R)\Big]$$

$$\mathrm{H}^{(\mathrm{K})}(\Phi_R) = \mathrm{R}(\Phi_R) - \mathrm{B}(\Phi_B)\,\mathrm{K}(\Phi_1)$$

- Parton shower described by generating functional $\mathcal{F}_{\mathrm{MC}}$

$$\langle O\rangle = \int \mathrm{d}\Phi_B\,\bar{\mathrm{B}}^{(\mathrm{K})}(\Phi_B)\,\mathcal{F}^{(0)}_{\mathrm{MC}}(\mu_Q^2,O) + \int \mathrm{d}\Phi_R\,\mathrm{H}^{(\mathrm{K})}(\Phi_R)\,\mathcal{F}^{(1)}_{\mathrm{MC}}(t(\Phi_R),O)$$

  Probability conservation: $\mathcal{F}_{\mathrm{MC}}(t,1) = 1 \to$ cross section correct at NLO

- Parametrically $\mathcal{O}(\alpha_s)$ correct, preserves logarithmic accuracy of PS

MC@NLO matching

- Insertion operators have simple analytic form

$$\mathbf{I} = -\frac{\alpha_s}{2\pi}\frac{1}{\Gamma(1-\varepsilon)}\left(\frac{4\pi\mu^2}{s_{ik}}\right)^{\varepsilon}\sum_{i,k\neq i}\frac{\mathbf{T}_{\tilde{i}}\mathbf{T}_k}{\mathbf{T}_{\tilde{i}}^2}\left(I_{i,k}^{\text{soft}} + I_{(ij)i}^{\text{coll}}\right)$$

- Soft

$$I_{i,k}^{\text{soft}} = \frac{1}{\varepsilon^2} + \frac{2}{\varepsilon} + 6 - \frac{\pi^2}{2} - 2\,\text{Re}\left\{\text{Li}_2\frac{1+\rho}{\rho}\right\} + 2\left(1+\rho+\log|\rho|\right)\log\frac{1+\rho}{\rho}$$
$$+ \text{Li}_2\left(1-\frac{\mu_K}{\rho\tau}\right) + \frac{1}{2}\ln^2\left(\frac{\rho}{\tau}\right) + (\text{asymmetric under } \rho \leftrightarrow \tau)$$

- Collinear

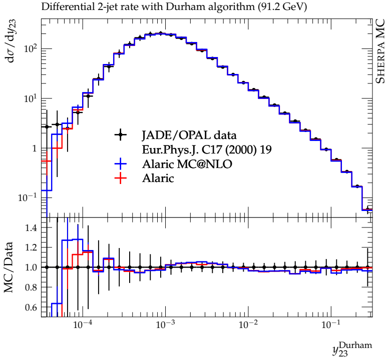
$$I_{qq}^{\text{coll}}(\hat{\kappa}) = -\frac{1}{2\varepsilon} - 1 - \frac{1}{2}\log\frac{s_{ik}}{Q^2} + \log(1-\hat{\kappa}) + \frac{1}{2}\frac{\hat{\kappa}\log\hat{\kappa}}{1-\hat{\kappa}}$$

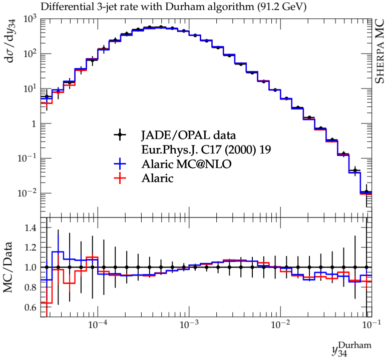
$$I_{gg}^{\text{coll}}(\hat{\kappa}) = -\frac{1}{6\varepsilon} - \frac{1}{6}\log\frac{s_{ik}}{Q^2} - \frac{8}{18}\frac{1-\hat{\kappa}/4}{1-\hat{\kappa}} - \frac{\hat{\kappa}^{3/2}}{3}\frac{\arcsin\sqrt{\hat{\kappa}}-\pi/2}{(1-\hat{\kappa})^{3/2}} + \frac{1}{3}\log(1-\hat{\kappa})$$

$$I_{gq}^{\text{coll}}(\hat{\kappa}) = -\frac{2}{3\varepsilon} - \frac{2}{3}\log\frac{s_{ik}}{Q^2} - \frac{16}{9}\frac{1-11\hat{\kappa}/8}{1-\hat{\kappa}} + \frac{2\hat{\kappa}^{3/2}}{3}\frac{\arcsin\sqrt{\hat{\kappa}}-\pi/2}{(1-\hat{\kappa})^{3/2}}$$
$$+ \frac{1}{3}\log(1-\hat{\kappa}) + \frac{\hat{\kappa}\log\hat{\kappa}}{1-\hat{\kappa}}\,.$$

$e^+e^- \rightarrow$ hadrons

[Krauss,Meinzinger,Reichelt,SH] TBP

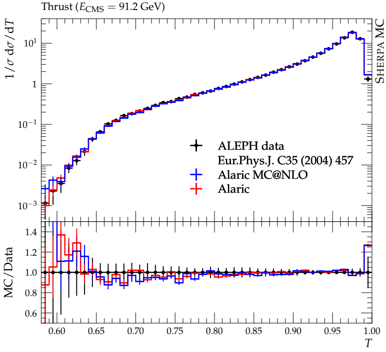


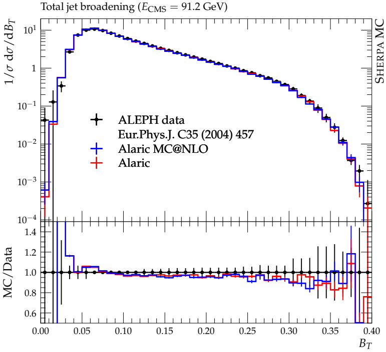


- Comparison to experimental data from LEP
- Radiation & splitting separated [Assi,SH] arXiv:2307.00728

$e^+e^- \rightarrow$hadrons

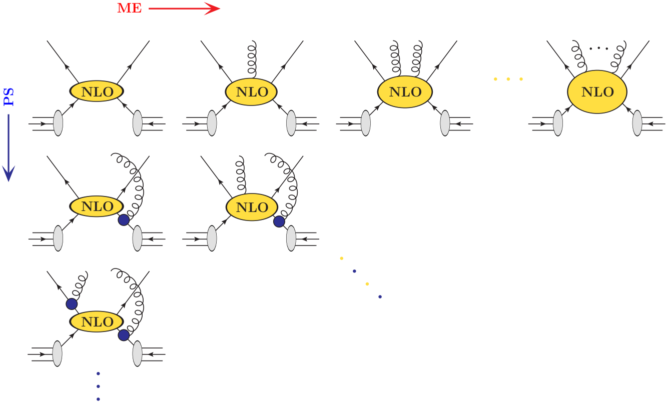
[Krauss,Meinzinger,Reichelt,SH] TBP





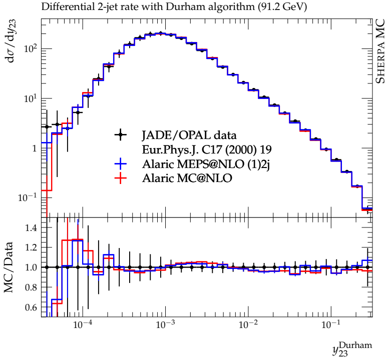
- Comparison to experimental data from LEP
- Radiation & splitting separated [Assi,SH] arXiv:2307.00728

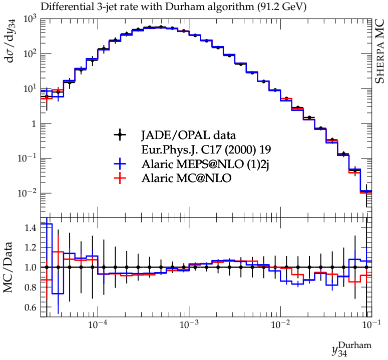
Next-to-leading order multi-jet merging



$e^+e^- \to$hadrons

[Krauss,Meinzinger,Reichelt,SH] TBP

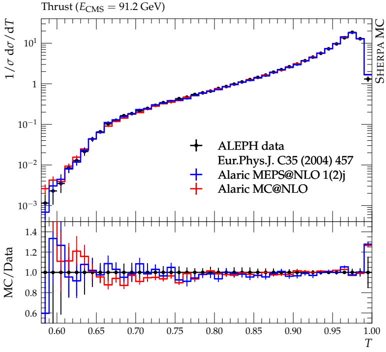


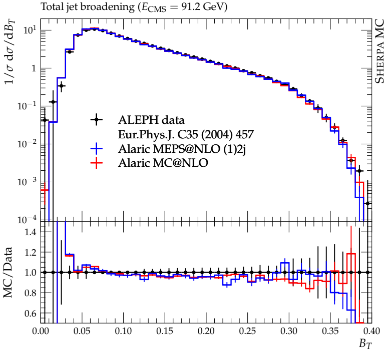


- Comparison to experimental data from LEP
- Radiation & splitting separated [Assi,SH] arXiv:2307.00728

$e^+e^- \to$ hadrons

[Krauss,Meinzinger,Reichelt,SH] TBP





- Comparison to experimental data from LEP
- Radiation & splitting separated [Assi,SH] arXiv:2307.00728

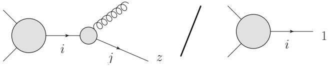
Towards fully differential NLO

Collinear evolution at NLO

- Higher-order DGLAP evolution kernels from factorization
  [Curci,Furmanski,Petronzio] NPB175(1980)27, [Floratos,Kounnas,Lacaze] NPB192(1981)417

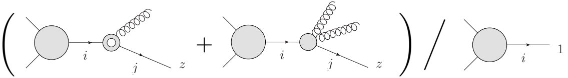
$$D_{ji}^{(0)}(z,\mu) = \delta_{ij}\delta(1-z) \qquad \leftrightarrow$$

$$D_{ji}^{(1)}(z,\mu) = -\frac{1}{\varepsilon}P_{ji}^{(0)}(z) \qquad \leftrightarrow$$



$$D_{ji}^{(2)}(z,\mu) = -\frac{1}{2\varepsilon}P_{ji}^{(1)}(z) + \frac{\beta_0}{4\varepsilon^2}P_{ji}^{(0)}(z) + \frac{1}{2\varepsilon^2}\int_z^1 \frac{\mathrm{d}x}{x}P_{jk}^{(0)}(x)P_{ki}^{(0)}(z/x)$$

$\leftrightarrow$



- In NLO parton shower, perform computation of $P_{ji}^{(1)}$ fully differentially using modified dipole subtraction [Catani,Seymour] hep-ph/9605323

Collinear evolution at NLO

[Prestel,SH] arXiv:1705.00742

- Schematically very similar to Catani-Seymour dipole subtraction
  e.g. simplest case of flavor-changing quark splitting

$$P^{(1)}_{qq'}(z) = \mathrm{C}_{qq'}(z) + \mathrm{I}_{qq'}(z) + \int \mathrm{d}\Phi_{+1} \Big[\mathrm{R}_{qq'}(z,\Phi_{+1}) - \mathrm{S}_{qq'}(z,\Phi_{+1})\Big]$$

  - Real correction $R_{qq'}$ and subtraction terms $S_{qq'}$
    given by $1 \to 3$ splitting and factorized expression
  - Integrated subtraction term and factorization counterterm

$$\mathrm{I}_{qq'}(z) = \int \mathrm{d}\Phi_{+1} S_{qq'}(z,\Phi_{+1})$$

$$\mathrm{C}_{qq'}(z) = \int_z \frac{\mathrm{d}x}{x} \left(P^{(0)}_{qg}(x) + \varepsilon \mathcal{J}^{(1)}_{qg}(x)\right) \frac{1}{\varepsilon} P^{(0)}_{gq}(z/x)$$

$$\mathcal{J}^{(1)}_{qg}(z) = 2C_F \left(\frac{1+(1-x)^2}{x} \ln(x(1-x)) + x\right)$$

- All components of $P^{(1)}_{ij}$ eventually finite in 4 dimensions
  Can be simulated fully differentially in parton shower

Soft evolution at NLO

[Catani,Grazzini] hep-ph/9908523

- Real-emission corrections can be written in convenient form

$$\mathcal{S}_{ij}^{(q\bar{q})}(1,2) = -\frac{s_{ij}}{(s_{i1}+s_{i2})(s_{j1}+s_{j2})}\frac{T_R}{s_{12}}\Big(1-4\,z_1 z_2\cos^2\phi_{12,ij}\Big)$$

$$\mathcal{S}_{ij}^{(gg)}(1,2) = \mathcal{S}_{ij}^{(\mathrm{s.o.})}(1,2)\,\frac{C_A}{2}\left(1+\frac{s_{i1}s_{j1}+s_{i2}s_{j2}}{(s_{i1}+s_{i2})(s_{j1}+s_{j2})}\right) + \frac{s_{ij}}{(s_{i1}+s_{i2})(s_{j1}+s_{j2})}\frac{C_A}{s_{12}}\Big(-2+4\,(1-\varepsilon)\,z_1 z_2\cos^2\phi_{12,ij}\Big)$$

- Strongly ordered and spin correlation components

$$\mathcal{S}_{ij}^{(\mathrm{s.o.})}(1,2) = \frac{s_{ij}}{s_{i1}s_{12}s_{j2}} + \frac{s_{ij}}{s_{j1}s_{12}s_{i2}} - \frac{s_{ij}^2}{s_{i1}s_{j1}s_{i2}s_{j2}}$$

$$4\,z_1 z_2\cos^2\phi_{12,ij} = \frac{(s_{i1}s_{j2}-s_{i2}s_{j1})^2}{s_{12}s_{ij}(s_{i1}+s_{i2})(s_{j1}+s_{j2})}$$

- Apparently simple structure, but unlike collinear NLO results not fully reflected by iterated leading-order splitting kernels

Soft evolution at NLO

[Dulat,Prestel,SH] arXiv:1805.03757

- After re-arrangement and addition of weight factors obtain a set of NLO-weighted LO splitting functions

$$(P_{qq})_i^k(1,2) = C_F\left(\frac{2\,s_{i2}}{s_{i1}+s_{12}}\,\frac{w_{ik}^{12}+\bar{w}_{ik}^{12}}{2}\right) + P_{ik}^{(\mathrm{slc})}(1,2)$$

$$(P_{gg})_{ij}(1,2) = C_A\left(\frac{2\,s_{i2}}{s_{i1}+s_{12}}\,\frac{w_{ij}^{12}+\bar{w}_{ij}^{12}}{2} + w_{ij}^{12}\Big(-1+z(1-z)\,2\cos^2\phi_{12}^{ij}\Big)\right)$$

$$(P_{gq})_{ij}(1,2) = T_R\,w_{ij}^{12}\Big(1-4z(1-z)\cos^2\phi_{12}^{ij}\Big)$$

- Additional subtracted real correction, virtuals & factorization counterterms Endpoint contributions given by

$$\tilde{\mathcal{S}}_{gq}^{(\mathrm{cusp})} = \delta(s_{12})\,\frac{2\,s_{ij}}{s_{i12}s_{j12}}\,T_R\Big[2z(1-z)+\big(1-2z(1-z)\big)\ln(z(1-z))\Big]$$

$$\tilde{\mathcal{S}}_{gg}^{(\mathrm{cusp})} = \delta(s_{12})\,\frac{2\,s_{ij}}{s_{i12}s_{j12}}\,2C_A\left[\frac{\ln z}{1-z}+\frac{\ln(1-z)}{z}+\big(-2+z(1-z)\big)\ln(z(1-z))\right]$$

$$\tilde{\mathcal{S}}_{wl}^{(\mathrm{cusp})} = -\,\delta(s_{i1})\,\frac{1}{2}\,\frac{C_A}{2}\,\frac{2\,s_{ij}}{s_{i12}s_{j12}}\left(\frac{\ln z_i}{1-z_i}+\frac{\ln(1-z_i)}{z_i}\right)+\Big(\text{swaps}\Big)$$

Sum integrates to CMW correction [Catani,Marchesini,Webber] NPB349(1991)635

Combination of soft and collinear expressions

Problems with existing splitting functions

- **Kinematical limits obscure underlying structure**
  Matching soft functions to collinear limit not straightforward
- **Different pQCD techniques for different limits**
  Soft limits in Feynman gauge, collinear ones in axial gauge

To understand the structure, we have to go back to basics
$\rightarrow$ recompute in common gauge and w/o taking limits

Say that again ... How can we NOT take limits?
It's the one thing we know how to do!

Combination of ~~soft~~ scalar and ~~collinear~~ splitting expressions

[Campbell,Höche,Knobbe,Preuss,Reichelt] arXiv:2505.10408

- Gordon decomposition [Gordon] ZeitPhys140(1928)630

$$\frac{\not{p}+\not{q}}{(p+q)^2}\,T^a_{ij}\gamma^\mu = T^a_{ij}\left[\,S^\mu(p,q)+\frac{i\sigma^{\nu\mu}q_\nu}{(p+q)^2}-\frac{\gamma^\mu\not{p}}{(p+q)^2}\,\right]$$

  - Leading and sub-leading (**LBK!**) soft behavior given by scalar current
    [Gell-Mann,Goldberger] PR96(1954)1433, [Brown,Goble] PR173(1968)1505

$$S^\mu(p,q)=\frac{(2p+q)^\mu}{(p+q)^2}$$

  - Magnetic term $\sigma^{\nu\mu}=i/2[\gamma^\nu,\gamma^\mu]$ due to quark spin
    $\gamma^\mu\not{p}$ generates seagull interactions of scalar theory
- Decomposition of triple & quartic gluon vertex even simpler (↗ Max' talk)
- Both decompositions hold at amplitude squared level [Chen et al.] arXiv:1404.5963
- **Separate scalar splitting functions & spin-dependent remainders**
  Clean identification of overlap beyond kinematical limits

Combination of ~~soft~~ scalar and ~~collinear~~ splitting expressions

[Campbell,Höche,Knobbe,Preuss,Reichelt] arXiv:2505.10408

- At 1-loop level, use Background Field Method [Abbott] NPB185(1981)189
- Allows to derive scalar radiators that satisfy naive Ward identities
  $\rightarrow$ extension of soft current in [Catani,Grazzini] hep-ph/0007142
- Clean decomposition of 1-loop splitting functions
  [Kosower,Uwer] hep-ph/9903515, [Bern,delDuca,Schmidt] hep-ph/9810409

Function	Scaling behavior for $\lambda \to 0$	
$\times s_{12}^{-1}$	$\tilde{p}_2 \to \lambda \tilde{p}_2$	$\tilde{p}_1 \to \lambda \tilde{p}_1$
$P^{(1)}_{\tilde{q}\to\tilde{q}}$	$\propto \lambda^{-2-2\epsilon}/\epsilon^2$	$\propto \lambda^{-\epsilon}/\epsilon^2$
$P^{(1,sc)}_{g\to g}$	$\propto \lambda^{-2-2\epsilon}/\epsilon^2$	$\propto \lambda^{-2\epsilon}/\epsilon^2$
$\langle P^{(1)}_{q\to q}\rangle$	$\propto \lambda^{-2-2\epsilon}/\epsilon^2$	$\propto \lambda^{-1-\epsilon}/\epsilon^2$
$\langle P^{(1,p)}_{q\to q}\rangle$	$\propto \lambda^{-2\epsilon}/\epsilon^2$	$\propto \lambda^{-1-\epsilon}/\epsilon^2$
$\langle P^{(1)}_{g\to g}\rangle$	$\propto \lambda^{-2-2\epsilon}/\epsilon^2$	
$\langle P^{(1,p)}_{g\to g}\rangle$	$\propto \lambda^{-2\epsilon}/\epsilon^2$	
$\langle P^{(1)}_{g\to q}\rangle$	$\propto \lambda^{-1-\epsilon}/\epsilon^2$	

Summary & Outlook

Current and future developments of Alaric:

- Higher-order corrections
  - Spin correlations ($\nearrow$ Mareen's talk)
  - Two-loop splitting functions ($\nearrow$ Max' talk)
- Fixed-order matching
  - MC@NLO for final-state evolution ($\nearrow$ Peter's talk)
  - Fully differential NNLO subtraction ($\nearrow$ Max' talk)
- Multi-jet merging
  - LO implementation completed
  - NLO for e^+e^- completed ($\nearrow$ Peter's talk)
- Practicalities
  - Release as part of Sherpa 3.1.x
  - Several components available in Python