

Challenges in MC Simulations: pp vs e^+e^-

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ITP Seminar

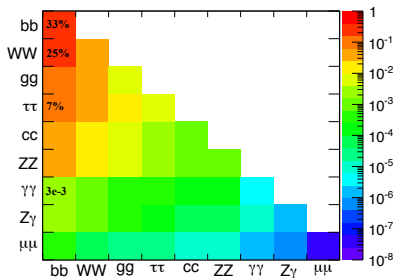
Göttingen, 10/09/2025

Outline

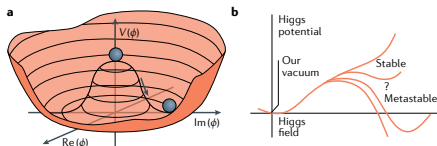
- A quick tour of LHC simulations
 - Big picture
 - Theory challenges
 - Lessons for FCC
- A tour of current FCC simulations
 - Big picture
 - Theory challenges
 - Lessons from LEP
- Towards higher precision
 - Perturbative QCD
 - QED / EW
 - Computing
- Needs and requirements

LHC – What we are preparing for

- Higgs self interaction is key to understanding of EW sector
- Measurement will require careful combination of many analyses with full HL-LHC data set
- Heavy flavor channels needed for high statistical significance



[J. Alison] LHCP '24



[Bass, DeRoeck, Kado] Nat. Rev. Phys. 3 (2021) 608

- Predictions for heavy quark production as part of inclusive heavy plus light flavor jets difficult to obtain at high precision
- Precise extraction of / limit setting on triple Higgs coupling depends crucially on understanding of all final states

Schematics of LHC simulations

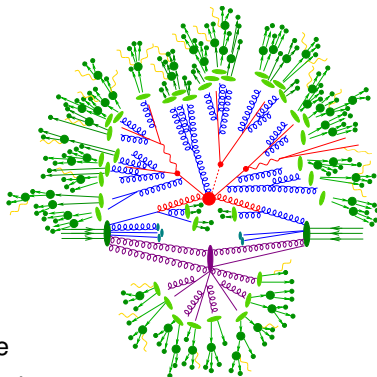
Need to cover large dynamic range

- Short distance interactions
 - **Signal process**
 - **Radiative corrections**
- Long-distance interactions
 - **Hadronization**
 - **Particle decays**

Divide and Conquer

- Quantity of interest: Total interaction rate
- Convolution of short & long distance physics

$$\sigma_{p_1 p_2 \rightarrow X} = \sum_{i,j \in \{q,g\}} \int dx_1 dx_2 \underbrace{f_{p_1,i}(x_1, \mu_F^2) f_{p_2,j}(x_2, \mu_F^2)}_{\text{long distance}} \underbrace{\hat{\sigma}_{ij \rightarrow X}(x_1 x_2, \mu_F^2)}_{\text{short distance}}$$



QCD theory as the primary tool

- $\hat{\sigma}_{ij \rightarrow n}(\mu_F^2) \rightarrow$ Collinearly factorized fixed-order result at N^xLO

Implemented in fully differential form to be maximally useful

Tree level: $d\Phi_n B_n$

- Automated ME generators + phase-space integrators

1-Loop level: $d\Phi_n \left(B_n + V_n + \sum C + \sum I_n \right) + d\Phi_{n+1} \left(R_n - \sum S_n \right)$

- Automated loop ME generators + integral libraries + IR subtraction

2-Loop level: It depends ...

- Individual solutions based on SCET, q_T subtraction, P2B

- $f_i(x, \mu_F^2) \rightarrow$ Collinearly factorized PDF at N^yLO

Evaluated at $O(1\text{GeV}^2)$ and expanded into a series above 1GeV^2

$$\text{DGLAP: } \frac{dx x f_a(x, t)}{d \ln t} = \sum_{b=q,g} \int_0^1 d\tau \int_0^1 dz \frac{\alpha_s}{2\pi} [z P_{ab}(z)]_+ \tau f_b(\tau, t) \delta(x - \tau z)$$

- Parton showers, dipole showers, antenna showers, ...

$$\text{Matching: } d\Phi_n \frac{S_n}{B_n} \leftrightarrow \frac{dt}{t} dz \frac{\alpha_s}{2\pi} P_{ab}(z)$$

- MC@NLO, POWHEG, Geneva, MINNLO_{PS}, ...

Directions of development

Much effort focused on perturbative QCD

- Phenomenologically interesting: Drives jet production, b -tagging, ...
- Experimentally relevant: Often source of largest uncertainty

Fixed-order aspects

- (N)NLO fixed order QCD
- Matching to parton shower
- Combination with QED (YFS)
- ... and NLO EW corrections

All-order aspects

- (N)NLL precision
- Heavy quark effects
- Sub-leading color & spin
- Threshold effects

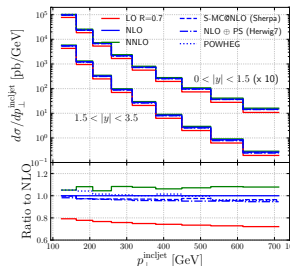
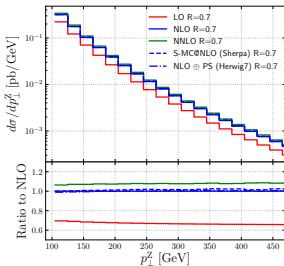
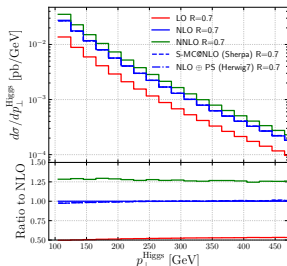
Understanding uncertainties & limitations

- Multi-year projects in context of LesHouches workshops to compare different generators on equal footing
- Growing community of MC devs & expert users in experiments with ties to MC groups & knowledge of common pitfalls in MC usage

Uncertainties in QCD NLO+PS matching

[Bellm et al.] arXiv:1903.12563

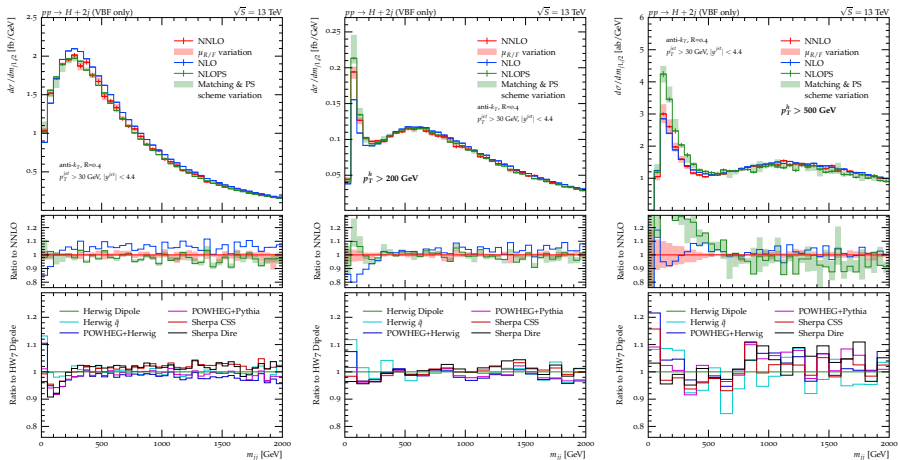
- Ratio of inclusive jet- $p_{\perp}$ cross sections for different radii in $pp \rightarrow jets$
- Excellent agreement of very different simulations $\rightarrow$ small uncertainties



Uncertainties in QCD NLO+PS matching

[Buckley et al.] arXiv:2105.11399

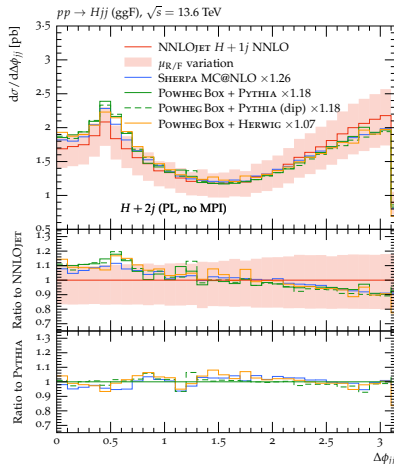
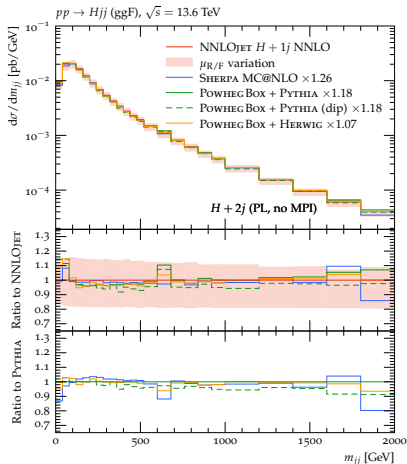
- m_{jj} of two leading jets in VBF Higgs production
- Excellent agreement of very different simulations → small uncertainties



Uncertainties in fixed-order + resummed simulations

[Chen et al.] arXiv:2509.10368

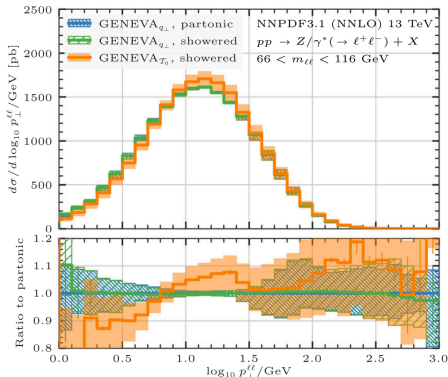
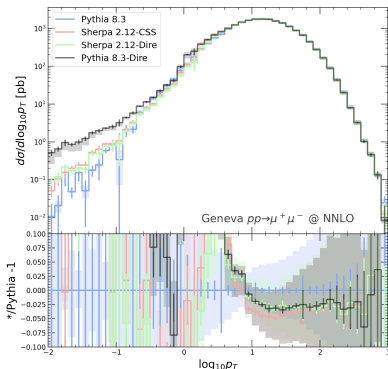
- Irreducible background to VBF Higgs boson production from gluon fusion
- Much smaller theoretical uncertainties ($\mathcal{O}(10\%)$) than estimated ($> 20\%$)



Uncertainties in QCD NNLO+PS matching

[D. Napoletano, HP2 2022], [Alioli et al.] arXiv:2102.08390

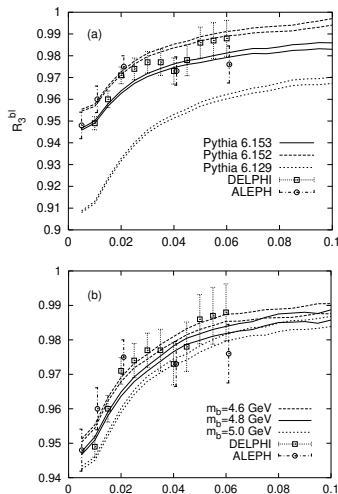
- NNLO+PS precise predictions for $pp \rightarrow Z$ from Geneva
- Matched to shower by vetoing events with $r_N(\Phi_{N+M}) > r_N$
- Significant residual uncertainties, even though NNLO



- Parton shower scheme uncertainty
- Choice of resolution variable

Heavy quark production

- Both high-energy limit and threshold region should be modeled as well as possible, but
- Infrared finite prediction for $g \rightarrow Q\bar{Q}$ leaves splitting functions somewhat arbitrary
- Soft gluon emission off light/heavy quarks associated with $\alpha_s(k_T^2)$, i.e. “correct” scale is k_T^2 [Amati et al.] NPB173(1980)429, but no such argument to set scale for $g \rightarrow Q\bar{Q} \rightarrow \text{HQ production rate}$ not very stable w.r.t. parton shower variations
- A number of different prescriptions, e.g.
[Norrbin,Sjöstrand], hep-ph/0010012,
[Gieseke,Stephens,Webber] hep-ph/0310083,
[Schumann,Krauss] arXiv:0709.1027,
[Gehrmann-deRidder,Ritzmann,Skands] arXiv:1108.6172
varying success in describing expt. data



[Norrbin,Sjöstrand] hep-ph/0010021

Heavy quark production

- Two different approaches to dealing with heavy-quark masses:
 - 4-flavor scheme (4FS): Decoupling scheme - (no b -quarks in PDF)
 - 5-flavor scheme (5FS): Minimal subtraction scheme

- Calculations can be matched by
 - Re-expressing both in same renormalization scheme
 - Subtracting the overlap

$$\sigma^{\text{FONLL}} = \sigma^{\text{massive}} + (\sigma^{\text{massless}} - \sigma^{\text{massive}, 0})$$

- This has been applied extensively to inclusive observables and is known as fixed-order next-to-leading log (FONLL) scheme

[Cacciari,Frixione,Mangano,Nason,Ridolfi] hep-ph/0312132,

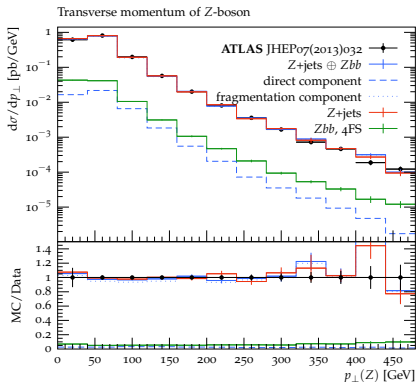
[Forte,Napoletano,Ubiali] arXiv:1508.01529, arXiv:1607.00389, . . .

- Extension to differential observables is needed for MC simulations
→ fully differential “fusing” algorithm [Krause,Siegert,SH] arXiv:1904.09382

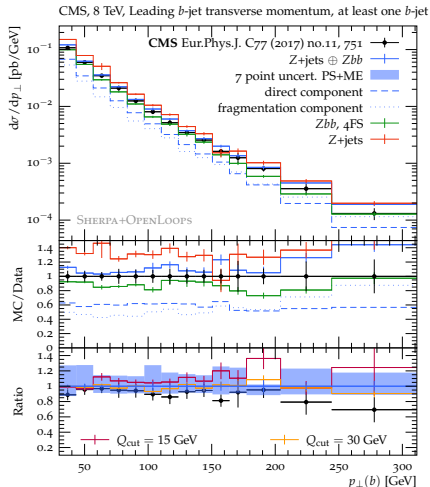
Heavy quark production

■ Z +jets vs $Zb\bar{b}$ at LHC

[Krause,Siegert,SH] arXiv:1904.09382



	Data [pb]	Fusing [pb]
$Z + \geq 1b$	$3.55 \pm 0.24_{\text{comb}}$	$3.80(5) \pm 0.83$ 0.33
$Z + \geq 2b$	$0.331 \pm 0.037_{\text{comb}}$	$0.282(4) \pm 0.027$ 0.022



Improvements needed for FCC

Fully differential high precision calculations

- NNLO QCD subtraction formalism
- Mixed QCD/EW corrections

Resummation and matching to fixed order

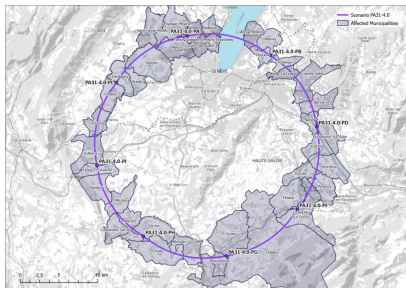
- Parton showers at NNLL precision
- Reduction of matching scheme uncertainty

Incorporation of quark mass effects

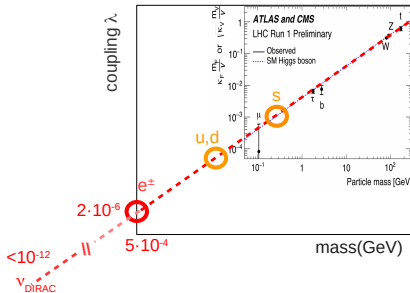
- Formal accuracy beyond FONLL-A
- Interplay with fragmentation functions

FCC – What we are preparing for

- Unprecedented luminosity at Tera-Z option of a potential FCC-ee would leave no room for mis-modeling of pQCD / npQCD or QED / EW effects



[CERN] <https://home.cern/science/accelerators/>



[D. d'Enterria] FCC week '24

- Extraction of Higgs Yukawa couplings would depend on precise modeling of light / heavy flavor jet production and flavor dynamics

Schematics of FCC simulations

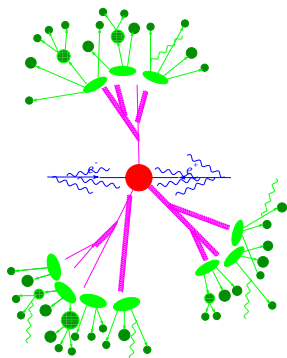
Need to cover modest dynamic range

- Short distance interactions
 - Signal process
 - QCD radiative corrections
 - QED radiative corrections
- Long-distance interactions
 - Hadronization
 - Particle decays

Divide and Conquer

- Quantity of interest: Interaction rate
- If hadrons involved, convolution of short & long distance physics, e.g.

$$\sigma_{p_1 p_2 \rightarrow X} = \sum_{i,j \in \{q,g\}} \int dx_1 dx_2 \underbrace{\hat{\sigma}_{ij+X}(x_1, x_2, \mu_F^2)}_{\text{short distance}} \underbrace{D_{h_1,i}(x_1, \mu_F^2) D_{h_2,j}(x_2, \mu_F^2) \dots}_{\text{long distance}}$$



Aspects of pQCD at FCC

Things to consider

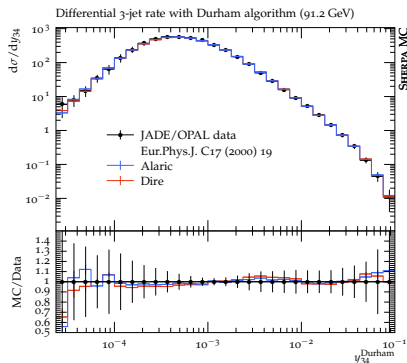
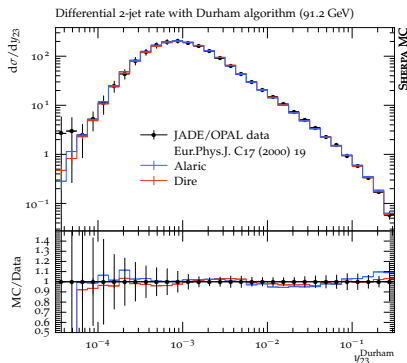
- At Tera-Z, the dynamic range is modest: $\sqrt{s} \approx 20 \times m_b$
QCD radiative effects are important, but still limited
We get about 7 gluons on average before hadronization
- This implies that understanding sub-leading powers is more important for precision than controlling higher logs
- Parton showers include some of those effects through exact phase-space & scalar splitting functions ($\nearrow$ later)

Consequences for MC development

- Parton-showers have to satisfy boundary conditions from analytic resummation, but we need to go beyond
- Much can be done by matching to fixed order, because the average number of emissions between $\sqrt{s}$ and Λ_{QCD} is small

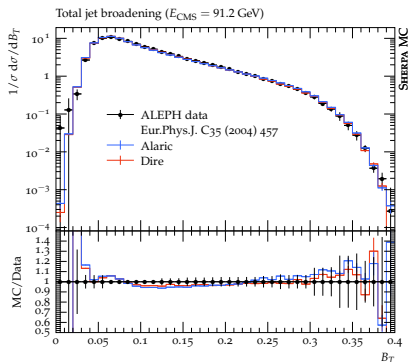
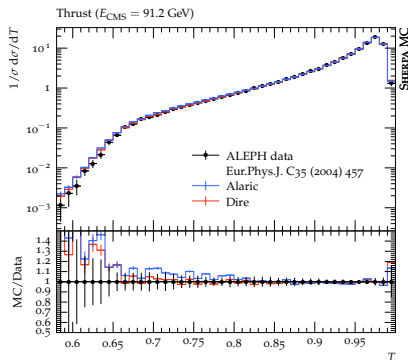
Typical performance of parton-showers

[Herren,Krauss,Reichelt,Schönherr,SH] arXiv:2208.06057



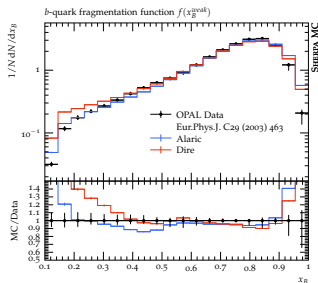
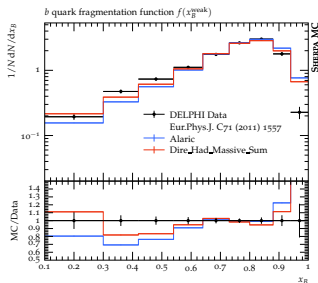
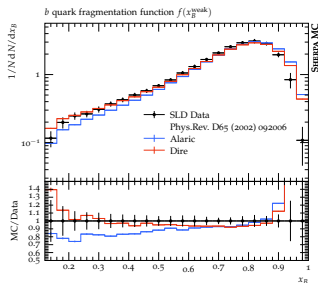
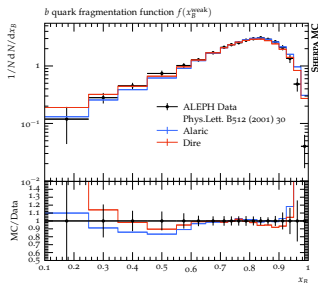
Typical performance of parton-showers

[Herren,Krauss,Reichelt,Schönherr,SH] arXiv:2208.06057



Typical performance in heavy quark evolution

[Assi,SH] arXiv:2307.00728



Impact of the momentum mapping

[Dasgupta,Dreyer,Hamilton,Monni,Salam] arXiv:1805.09327

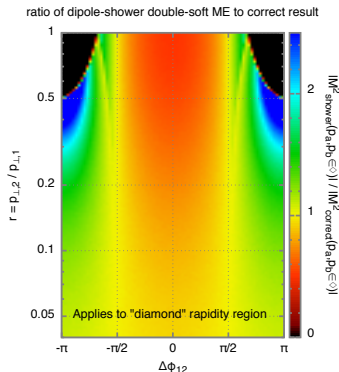
- Some dipole-like momentum mappings violate strong ordering approximation

$$p_k^\mu = \left(1 - \frac{p_{ij}^2}{2\tilde{p}_{ij}\tilde{p}_k}\right) \tilde{p}_k^\mu$$

$$p_i^\mu = \tilde{z} \tilde{p}_{ij}^\mu + (1 - \tilde{z}) \frac{p_{ij}^2}{2\tilde{p}_{ij}\tilde{p}_k} \tilde{p}_k^\mu + k_\perp^\mu$$

$$p_j^\mu = (1 - \tilde{z}) \tilde{p}_{ij}^\mu + \tilde{z} \frac{p_{ij}^2}{2\tilde{p}_{ij}\tilde{p}_k} \tilde{p}_k^\mu - k_\perp^\mu$$

- Angular correlations across multiple emissions due to recoil on splitter in anti-collinear region
- Spoils $\alpha_s \rightarrow 0$ consistency check
 $\leftrightarrow$ NLL accuracy cannot be achieved



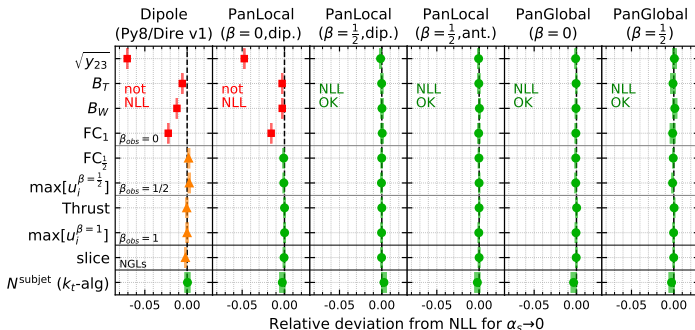
Impact of the momentum mapping

[Dasgupta,Dreyer,Hamilton,Monni,Salam,Soyez] arXiv:2002.11114

- Problem can be solved e.g. by partitioning of antenna radiation pattern and choosing a suitable evolution variable ($\beta \sim 1/2$)

$$k_T = \rho v e^{\beta |\vec{\eta}|} \quad \rho = \left(\frac{s_i s_j}{Q^2 s_{ij}} \right)^{\beta/2}$$

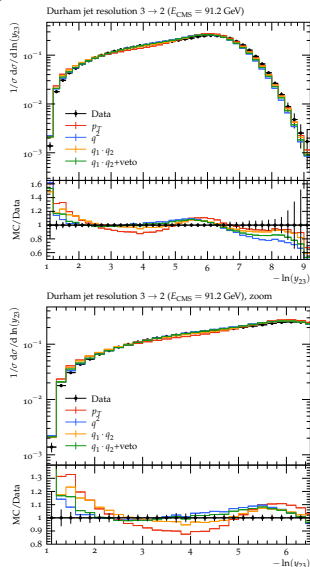
- NLL correct for global and non-global observables in $e^+e^- \rightarrow \text{hadrons}$



Impact of the momentum mapping

[Bewick,Ferrario-Ravasio,Richardson,Seymour] arXiv:1904.11866

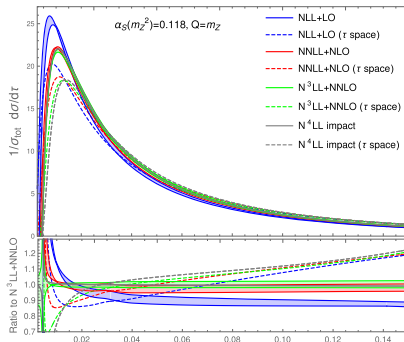
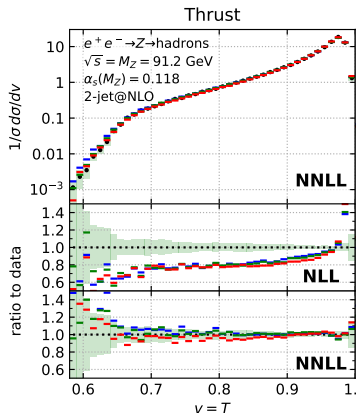
- Note: Recoil schemes affect logarithmic accuracy but impact also phase-space coverage & sub-leading power effects
- In context of angular ordered Herwig 7 (NLL accurate for global observables)
 - q_T preserving scheme:
 - Maintains logarithmic accuracy
 - Overpopulates hard region
 - q^2 preserving scheme:
 - Breaks logarithmic accuracy
 - Good description of hard region
 - Dot product preserving scheme (new):
 - Maintains logarithmic accuracy
 - Good description of hard radiation



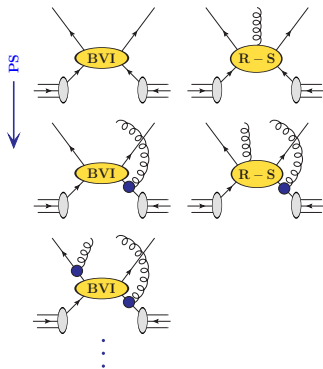
NNLL evolution for event shapes

[vanBeekveld et al.] arXiv:2406.02661

- NNLL for global event shapes achieved recently
- Found differences of $\mathcal{O}(20\%)$ between NLL and NNLL
Compare to analytic computation [Aglietti,Ferrera,Ju,Miao] arXiv:2502.01570
- Better understanding needed to achieve target precision for FCC-ee
→ Is there a need for N^3 LL, or rather sub-leading power?

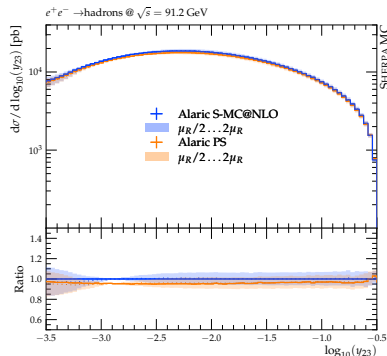
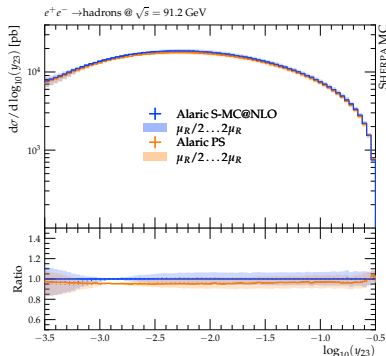


MC@NLO matching



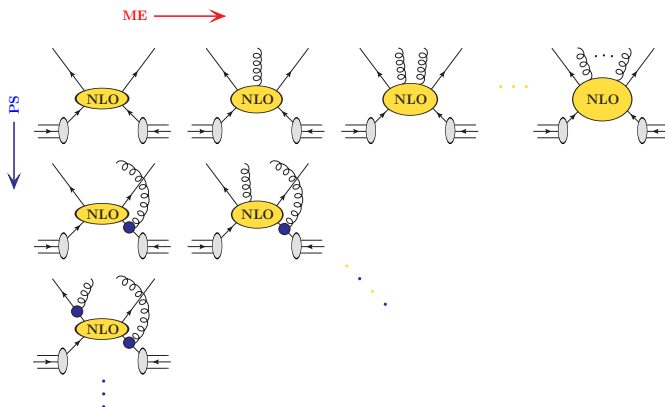
Typical performance of MC@NLO matching

[Krauss,Meinzinger,Reichelt,SH] arXiv:2507.22837



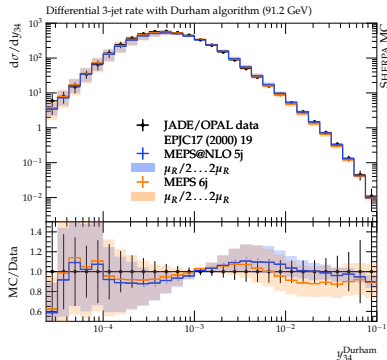
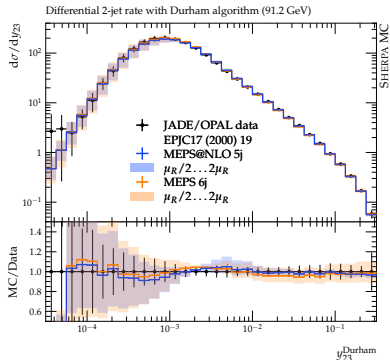
- Jet rates in Durham algorithm
- Radiation pattern determined almost exclusively by PS

Multi-jet merging



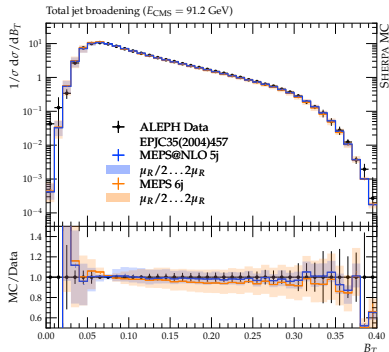
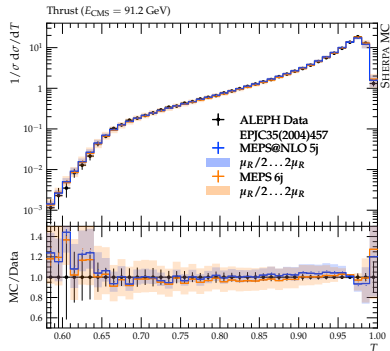
Typical performance of matching & merging

[Krauss,Meinzinger,Reichelt,SH] arXiv:2507.22837



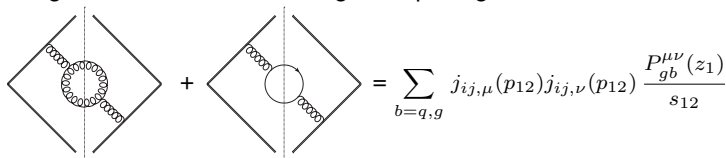
Typical performance of matching & merging

[Krauss,Meinzinger,Reichelt,SH] arXiv:2507.22837



Towards NLO QCD evolution: Soft limit

- Approximate soft-gluon emission times collinear decay in $q(i)\bar{q}(j)g(1)g(2)$ using semi-classical limit and gluon splitting function



$$\text{Diagram 1} + \text{Diagram 2} = \sum_{b=q,g} j_{ij,\mu}(p_{12})j_{ij,\nu}(p_{12}) \frac{P_{gb}^{\mu\nu}(z_1)}{s_{12}}$$

$$P_{gq}^{\mu\nu}(z) = T_R \left(-g^{\mu\nu} + 4z(1-z) \frac{k_{\perp}^{\mu} k_{\perp}^{\nu}}{k_{\perp}^2} \right)$$

$$P_{gg}^{\mu\nu}(z) = C_A \left(-g^{\mu\nu} \left(\frac{z}{1-z} + \frac{1-z}{z} \right) - 2(1-\varepsilon)z(1-z) \frac{k_{\perp}^{\mu} k_{\perp}^{\nu}}{k_{\perp}^2} \right)$$

- Combine with phase space for one parton emission in collinear limit

$D = 4 - 2\varepsilon$, $y = s_{12}/Q^2$, see for example [Catani,Seymour] hep-ph/9605323

$$d\Phi_{+1} = \frac{Q^{2-2\varepsilon}}{16\pi^2} \frac{(4\pi)^{\varepsilon}}{\Gamma(1-\varepsilon)} dy dz [yz(1-z)]^{-\varepsilon}$$

- Perform Laurent series expansion

$$\frac{1}{y^{1+\varepsilon}} = -\frac{\delta(y)}{\varepsilon} + \sum_{n=0}^{\infty} \frac{\varepsilon^n}{n!} \left(\frac{\ln^n y}{y} \right)_+$$

Towards NLO QCD evolution: Soft limit

- $\mathcal{O}(\varepsilon^0)$ differential remainder terms have contributions proportional to

$$g \rightarrow q\bar{q}: T_R \left[2z(1-z) + (1-2z(1-z)) \ln(z(1-z)) \right]$$

$$g \rightarrow gg: 2C_A \left[\frac{\ln z}{1-z} + \frac{\ln(1-z)}{z} + (-2 + z(1-z)) \ln(z(1-z)) \right]$$

- Integration over z , addition of some semi-classical terms & one-loop soft current gives two-loop cusp anomalous dimension

$$K = \left(\frac{67}{18} - \frac{\pi^2}{6} \right) C_A - \frac{10}{9} T_R n_f$$

- Local K -factor for soft-gluon emission
 - Scheme dependent: originates in dim. reg. and $\overline{\text{MS}}$
 - **Can be absorbed in effective coupling** [Catani, Marchesini, Webber] NPB349(1991)635
- Similarly, we find $\mathcal{O}(\varepsilon^0)$ contributions proportional to

$$\frac{\alpha_s}{2\pi} \beta_0 \log \frac{(p_i p_{12})(p_{12} p_j)}{(p_i p_j) \mu^2}$$

- Can be eliminated by setting scale to transverse mass of soft pair
- **Leading NLO correction** [Amati, et al.] NPB173(1980)429

Towards NLO QCD evolution: Collinear limit

■ Higher-order DGLAP evolution kernels from factorization

[Curci,Furmanski,Petronzio] NPB175(1980)27, [Floratos,Kounnas,Lacaze] NPB192(1981)417

$$D_{ji}^{(0)}(z, \mu) = \delta_{ij} \delta(1-z) \quad \leftrightarrow \quad \text{Diagram 1} / \text{Diagram 2}$$

$$D_{ji}^{(1)}(z, \mu) = -\frac{1}{\epsilon} P_{ji}^{(0)}(z) \quad \leftrightarrow \quad \text{Diagram 3} / \text{Diagram 4}$$

$$D_{ji}^{(2)}(z, \mu) = -\frac{1}{2\epsilon} P_{ji}^{(1)}(z) + \frac{\beta_0}{4\epsilon^2} P_{ji}^{(0)}(z) + \frac{1}{2\epsilon^2} \int_z^1 \frac{dx}{x} P_{jk}^{(0)}(x) P_{ki}^{(0)}(z/x)$$

$$\leftrightarrow \left(\text{Diagram 5} + \text{Diagram 6} \right) / \text{Diagram 7}$$

The diagrams represent the collinear limits of the DGLAP evolution kernels. Diagram 1 is a circle with two external lines, labeled j and i , and a momentum fraction z . Diagram 2 is a circle with two external lines, labeled i and 1 . Diagram 3 is a circle with two external lines, labeled i and j , and a momentum fraction z , with a wavy line (gluon) attached to the circle. Diagram 4 is a circle with two external lines, labeled i and 1 . Diagram 5 is a circle with two external lines, labeled i and j , and a momentum fraction z , with a wavy line (gluon) attached to the circle. Diagram 6 is a circle with two external lines, labeled i and j , and a momentum fraction z , with a wavy line (gluon) attached to the circle. Diagram 7 is a circle with two external lines, labeled i and 1 .

- In NLO parton shower, perform computation of $P_{ji}^{(1)}$ fully differentially using modified dipole subtraction [Catani,Seymour] hep-ph/9605323

Towards NLO QCD evolution: Collinear limit

[Prestel,SH] arXiv:1705.00742

- Schematically very similar to Catani-Seymour dipole subtraction
e.g. simplest case of flavor-changing quark splitting

$$P_{qq'}^{(1)}(z) = C_{qq'}(z) + I_{qq'}(z) + \int d\Phi_{+1} \left[R_{qq'}(z, \Phi_{+1}) - S_{qq'}(z, \Phi_{+1}) \right]$$

- Real correction $R_{qq'}$ and subtraction terms $S_{qq'}$
given by $1 \rightarrow 3$ splitting and factorized expression
- Integrated subtraction term and factorization counterterm

$$I_{qq'}(z) = \int d\Phi_{+1} S_{qq'}(z, \Phi_{+1})$$

$$C_{qq'}(z) = \int_z \frac{dx}{x} \left(P_{qg}^{(0)}(x) + \varepsilon \mathcal{J}_{qg}^{(1)}(x) \right) \frac{1}{\varepsilon} P_{gq}^{(0)}(z/x)$$

$$\mathcal{J}_{qg}^{(1)}(z) = 2C_F \left(\frac{1 + (1-x)^2}{x} \ln(x(1-x)) + x \right)$$

- All components of $P_{ij}^{(1)}$ eventually finite in 4 dimensions
Can be simulated fully differentially in parton shower

Combination of soft and collinear expressions

Problems with existing splitting functions

- **Kinematical limits obscure underlying structure**

Matching soft functions to collinear limit not straightforward

- **Different pQCD techniques for different limits**

Soft limits in Feynman gauge, collinear ones in axial gauge

To understand the structure, we have to go back to basics

→ recompute in common gauge and w/o taking limits

Say that again ... How can we NOT take limits?

It's the one thing we know how to do!

Combination of ~~soft~~ scalar and ~~collinear~~ splitting expressions

[Campbell,Knobbe,Preuss,Reichelt,SH] arXiv:2505.10408

- Gordon decomposition [Gordon] ZeitPhys140(1928)630

$$\frac{\not{p} + \not{q}}{(p+q)^2} T_{ij}^a \gamma^\mu = T_{ij}^a \left[S^\mu(p, q) + \frac{i\sigma^{\nu\mu} q_\nu}{(p+q)^2} - \frac{\gamma^\mu \not{p}}{(p+q)^2} \right]$$

- Leading and sub-leading (**LBK!**) soft behavior given by scalar current

[Gell-Mann,Goldberger] PR96(1954)1433, [Brown,Goble] PR173(1968)1505

$$S^\mu(p, q) = \frac{(2p+q)^\mu}{(p+q)^2}$$

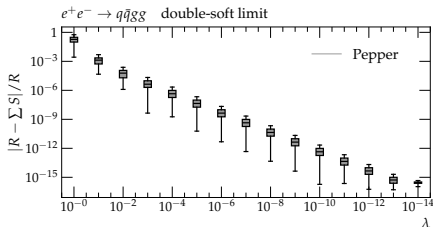
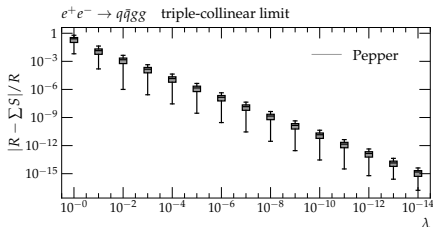
- Magnetic term $\sigma^{\nu\mu} = i/2[\gamma^\nu, \gamma^\mu]$ due to quark spin
 $\gamma^\mu \not{p}$ generates seagull interactions of scalar theory

- Decomposition of triple & quartic gluon vertex even simpler
- Both decompositions hold at amplitude squared level [Chen et al.] arXiv:1404.5963
- **Separate scalar splitting functions & spin-dependent remainders**
Clean identification of overlap beyond kinematical limits
- At 1-loop level, Background Field Method allows to derive
Scalar radiators that satisfy the naive Ward identities
→ Extension of soft current [Catani,Grazzini] hep-ph/0007142

Application to NNLO fixed-order calculations

[M. Knobbe] PSR'25, QCD@LHC'25, [Knobbe,SH] WIP

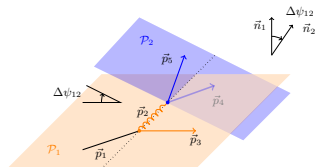
- *Novel infrared subtraction for NNLO calculations* currently under development
- No overlap between scalar and splitting components
→ straightforward assembly of complete IR counterterms
- Suitable for matching to a fully differential resummation at NLO QCD precision (first components of which in [Prestel,SH] arXiv:1705.00742 & [Dulat,Prestel,SH] arXiv:1805.03757)



Application to parton showers

[Hoppe,Reichelt,SH] arXiv:2508.19018

- Conventional wisdom: Gluon spin correlations are a quantum effect



[Chen,Moult,Zhu] arXiv:2011.02492

[Karlberg,Salam,Scyboz,Verheyen]

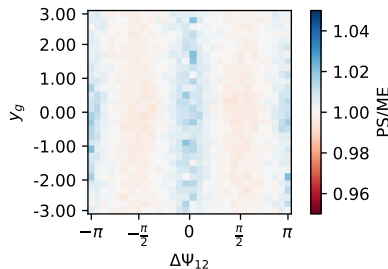
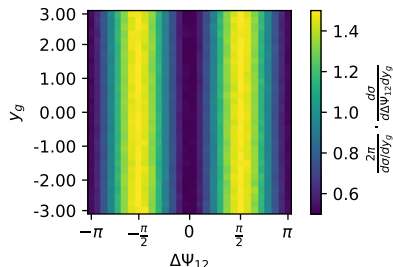
arXiv:2103.15526

- Re-analyze using new formalism
→ Most correlations are classical

[Staelin,Morgenthaler,Kong]

Electromagnetic Waves, Pearson (1993)

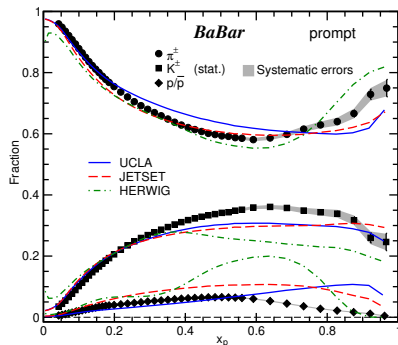
- Reproduced perfectly in simulation using simple and robust algorithm



The Need for improved Hadronization Models

- Modeling of non-perturbative parton-to-hadron transition important for detector response, especially at low particle multiplicity
- Flavor composition of jets and identified hadron production typically challenging to model, especially at low energy
- Must be addressed in order to reach precision goals of FCC-ee

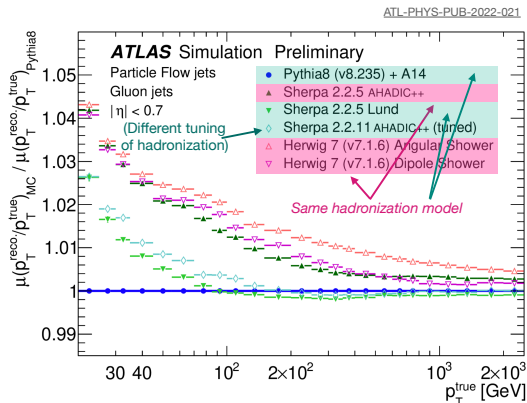
[Lees et al.] arXiv:1306.2895



The Need for improved Hadronization Models

[J. Roloff, PSR'25]

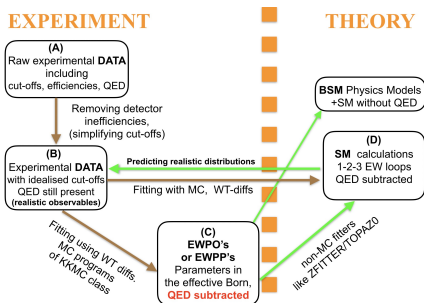
- ▶ Obvious trend in the jet response from the hadronization model
- ▶ *Calorimeter response depends on the type of hadron, not just the energy and rapidity*
- ▶ Using **retuned Sherpa** with LEP data on baryon and kaon fractions has a significant effect on the p_T response!



The Need for Precise QED Simulations

[Jadach,Skrzypek] arXiv:1903.09895

- Projected $2\text{--}100\times$ improvement in measurement of EWPOs
- Per mille-level uncertainties could be ignored at LEP but not at FCC-ee, particularly Tera-Z option
- QED radiative effects must be modeled as precisely as possible



Observable	Where from	Present (LEP)	FCC stat.	FCC syst	Now FCC
M_Z [MeV]	Z linesh.	$91187.5 \pm 2.1\{0.3\}$	0.005	0.1	3
Γ_Z [MeV]	Z linesh.	$2495.2 \pm 2.1\{0.2\}$	0.008	0.1	2
$R_l^Z = \Gamma_h/\Gamma_l$	$\sigma(M_Z)$	$20.767 \pm 0.025\{0.012\}$	$6 \cdot 10^{-5}$	$1 \cdot 10^{-3}$	12
σ_{had}^0 [nb]	σ_{had}^0	$41.541 \pm 0.037\{0.025\}$	$0.1 \cdot 10^{-3}$	$4 \cdot 10^{-3}$	6
N_ν	$\sigma(M_Z)$	$2.984 \pm 0.008\{0.006\}$	$5 \cdot 10^{-6}$	$1 \cdot 10^{-3}$	6
N_ν	$Z\gamma$	$2.69 \pm 0.15\{0.06\}$	$0.8 \cdot 10^{-3}$	$< 10^{-3}$	60
$\sin^2 \theta_{W}^{eff} \times 10^5$	$A_{FB}^{lept.}$	$23099 \pm 53\{28\}$	0.3	0.5	55
$\sin^2 \theta_{W}^{pol} \times 10^5$	$\langle P_\tau \rangle, A_{FB}^{pol, \tau}$	$23159 \pm 41\{12\}$	0.6	< 0.6	20
M_W [MeV]	ADLO	$80376 \pm 33\{6\}$	0.5	0.3	12
$A_{FB, \mu}^{M_Z \pm 3.5 \text{ GeV}}$	$\frac{d\sigma}{d\cos\theta}$	$\pm 0.020\{0.001\}$	$1.0 \cdot 10^{-5}$	$0.3 \cdot 10^{-5}$	100

QED Resummation

Collinear Resummation

[Frixione et al.] JHEP03(2020)135

- Collinear logs are resummed with universal PDF
- Matched to NLO_{EW}
- Combined with Parton Shower to generate photon emissions
- Beyond NLO becomes tricky

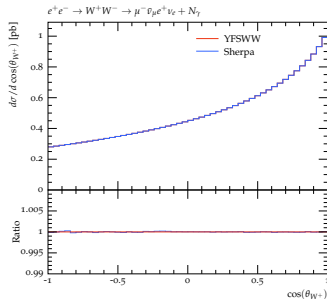
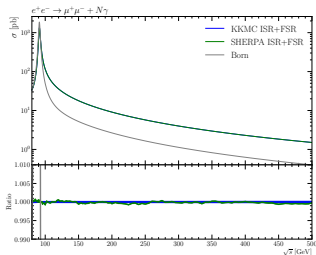
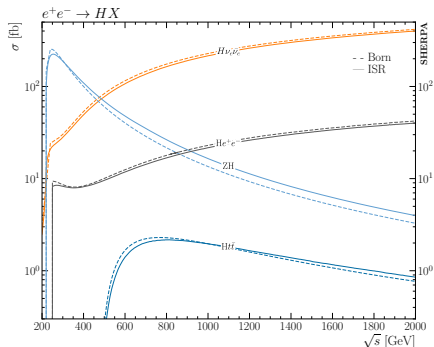
Soft Resummation

[Jadach et.al] ZPC49(1991)577, EPL17(1992)123

- Soft logs resummed to infinite order using the YFS method
[Yennie,Frautschi,Suura] Ann.Phys.13(1961)379
- Provides a robust scheme for the inclusion of real and virtual corrections at any order.
- Collinear terms can be added

YFS Resummation

- Modern YFS tools validated carefully against state-of-the-art from LEP, e.g. KKMC [Jadach,Ward,Was] hep-ph/9912214, YFSWW [Jadach et al.] hep-ph/0104049



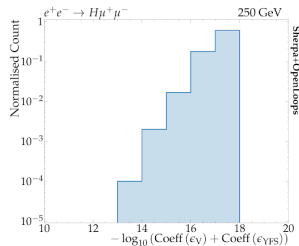
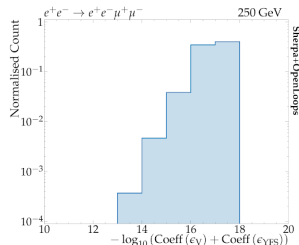
YFS Matching: Virtual corrections

$$\tilde{\beta}_0^1(\Phi_n) = \mathcal{V}(\Phi_n) - \sum_{ij} \mathcal{D}_{ij}(\Phi_{ij})$$

Diagram illustrating the equation for the full one-loop amplitude:

- $\tilde{\beta}_0^1(\Phi_n)$ is labeled as the **IR Finite one-loop contribution**.
- $\mathcal{V}(\Phi_n)$ is labeled as the **Full One-loop amplitude**.
- $\mathcal{D}_{ij}(\Phi_{ij})$ is labeled as the **YFS Subtraction term**.

- Expansion of resummed soft virtual in YFS provides IR counterterms for exact virtual
- Pole cancellation occurs locally in phase space $\rightarrow$ checks against 1-loop



YFS Matching: Real corrections

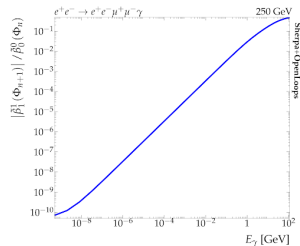
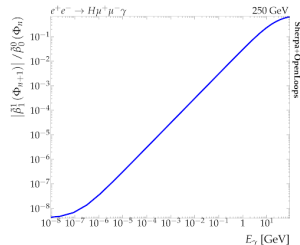
$$\tilde{\beta}_1^1(\Phi_{n+1}) = \frac{1}{2(2\pi)^3} \left| \mathcal{M}_0^{\frac{1}{2}}(\Phi_{n+1}) \right|^2 - \sum_{ij} \tilde{\mathcal{D}}_{ij}(\Phi_{ij+1} \otimes \Phi_n)$$

IR Finite contribution for real corrections

Real emission squared amplitude

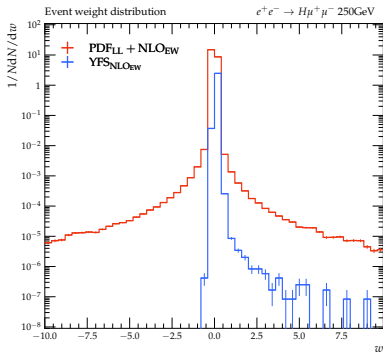
YFS Subtraction term

- Expansion of resummed soft real in YFS provides IR counterterms for exact real



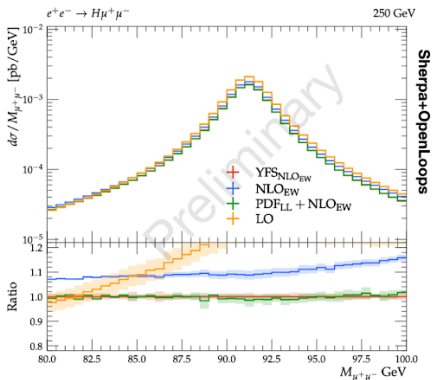
YFS Matching: Efficiency

- Tera-Z will require a significantly larger number of events to be generated than LHC
- Statistical power of event sample quickly becomes an issue
- YFS produces significantly smaller fraction of negative weighted events than e.g. CS subtraction

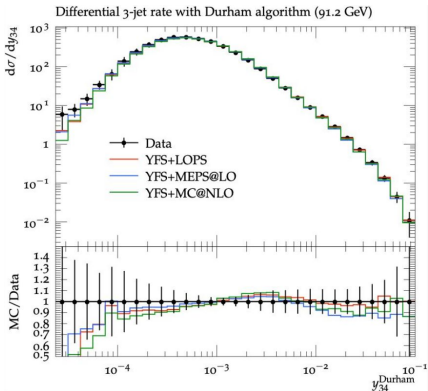


YFS Case Studies

Higgs-Strahlung at FCC



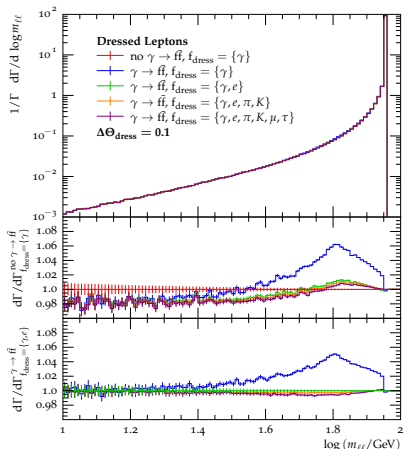
QED ISR effects at the Z pole



Photon splittings

[Flower,Schönherr] arXiv:2210.07007

- YFS resummation does not include $\gamma \rightarrow f\bar{f}$ corrections, which enter at $\mathcal{O}(\alpha^2) \rightarrow$ must be added explicitly
- Evolve in parton-shower picture until photon virtuality drops below $m_{f\bar{f}}$
- Dress additional charges with additional photons using YFS



Efficient Computing

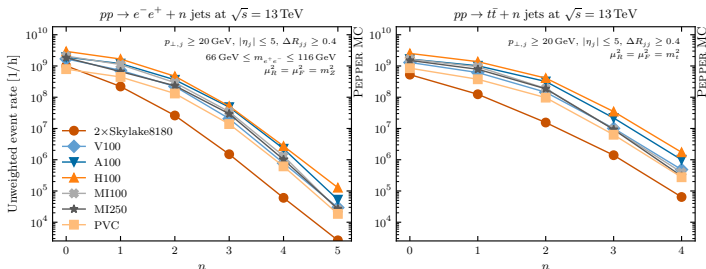
- Modern computing → many vendors & heterogeneous architectures
- (Pre-)Exascale computing systems intentionally diverse



Efficient Computing

[Bothmann et al.] arXiv:2311.06198

- Performance portability a major topic of discussion
Driven by trends in computing industry and development at LCFs
- Portability of new codes achieved by Kokkos

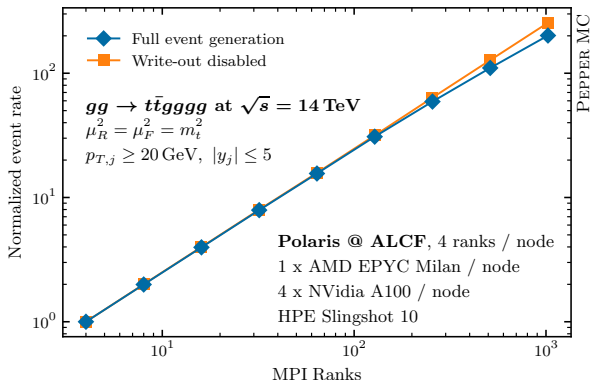


MEvents / hour	2xSkylake8180	V100	A100	H100	MI100	MI250	PVC
$pp \rightarrow t\bar{t} + 4j$	0.06	0.5	1.0	1.7	0.4	0.3	0.3
$pp \rightarrow e^-e^+ + 5j$	0.003	0.03	0.05	0.1	0.03	0.03	0.02

Efficient Computing

[Bothmann et al.] arXiv:2311.06198, arXiv:2309.13154

- Scalability achieved for up to $1024 \times A100$'s (1/2 of Polaris)
- Scaling violations only due to I/O, no issue for few nodes / low data volume
Will have to be assessed once computing model for Tera-Z has taken shape



Summary & Discussion

- Perturbative QCD on track to deliver sufficient precision for FCC-ee
Physics performance likely limited by understanding of hadronization
- Tera-Z will require highest MC statistics of any experiment so far
May only be achievable with the help of HPC, possibly LCFS
- EWPOs will require multi-loop QED / EW calculations
Must be implemented in MCs, at least partially
- Some of these developments overlap with LHC, some do not
Mechanism needed for WFD and retention of talented developers