Precision simulations of light and heavy jets

Stefan Höche

Fermi National Accelerator Laboratory

Theory Seminar

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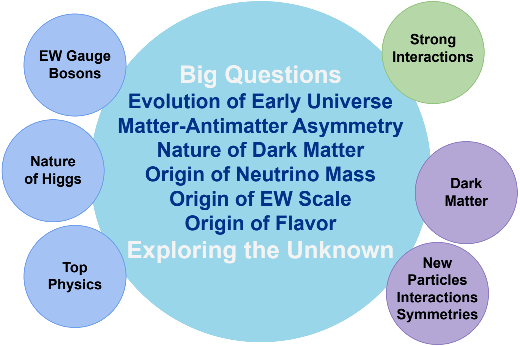
The future of the Energy Frontier

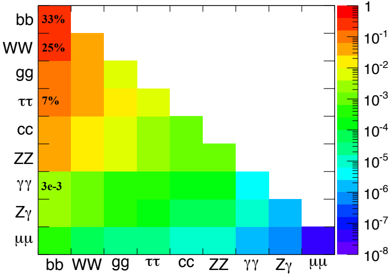
[Narain et al.] arXiv:2211.11084



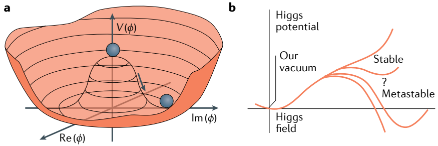
- What can we learn about the origin of the EW scale and the EW phase transition from an in-depth study of SM particles at colliders (HL-LHC)?
- What can we learn about the dynamics of strong interactions?
- How can we build a complete program of BSM searches which includes both model-specific and model-independent explorations at high scales?

The future of the Energy Frontier

- Higgs self interaction is key to understanding of EW sector
- Measurement will require careful combination of many analyses with full HL-LHC data set
- Heavy flavor channels needed for high statistical significance



[J. Alison] LHCP ’24

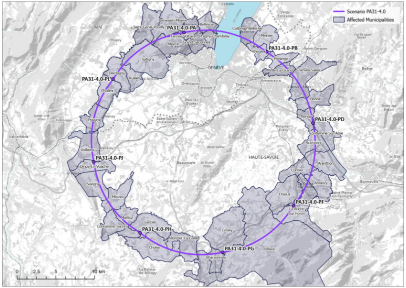


[Bass, DeRoeck, Kado] Nat. Rev. Phys. 3 (2021) 608

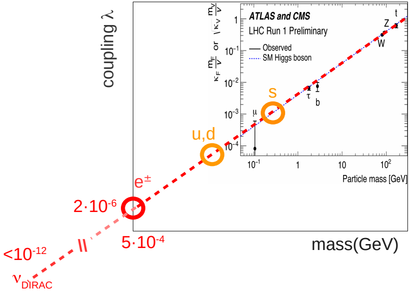
- Predictions for heavy quark production as part of inclusive heavy plus light flavor jets difficult to obtain at high precision
- Precise extraction of / limit setting on triple Higgs coupling depends crucially on understanding of all final states

The future of the Energy Frontier

- Unprecedented luminosity at Tera-Z option of a potential FCC-ee will leave no room for mis-modeling of non-perturbative QCD effects



[CERN] https://home.cern/science/accelerators/

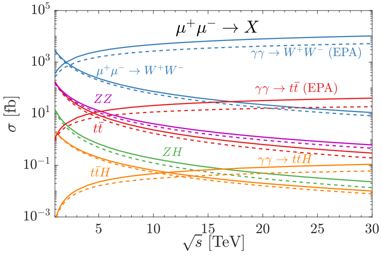


[D. d'Enterria] FCC week '24

- Extraction of Higgs Yukawa couplings will depend on precise modeling of light / heavy flavor jet production and flavor dynamics

The future of the Energy Frontier

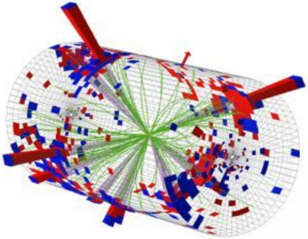
- New collider concepts require different theoretical and computational strategies
- At highest energies targeted by muon collider concepts, electroweak sector of Standard Model requires resummation

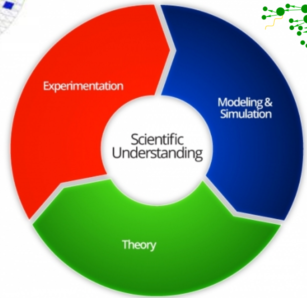


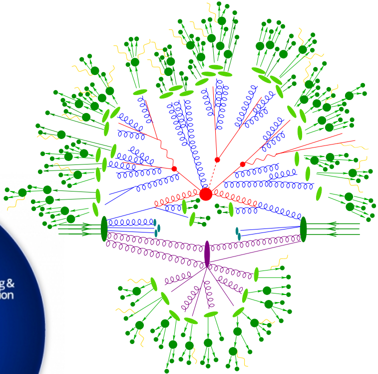
[Han,Ma,Xie] arXiv:2007.14300

[Science] March ’24

How to explore the unknown



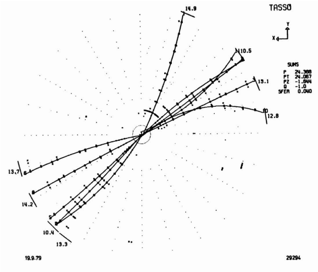


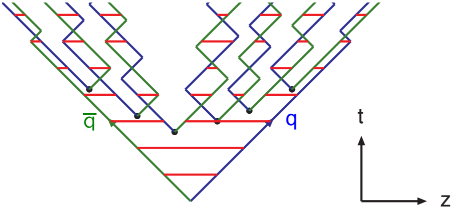


$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i \bar{\psi} \not{D} \psi + h.c.$$

50 years ago: Di-jet events at PETRA

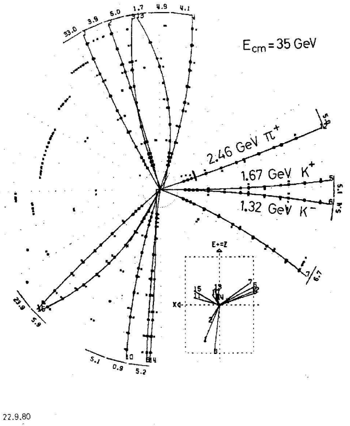
[Andersson,Gustafson,Ingelman,Sjöstrand] Phys.Rept.97(1983)31





- Lund string model: QCD flux tube like a rubber band that is pulled apart $\rightarrow$ breaks into pieces, generating many smaller flux tubes.
- Creates two collimated sprays of hadrons $\rightarrow$ 2-jet events
- Complete description of the physics at low-energy e^+e^--colliders

The gluon changes everything



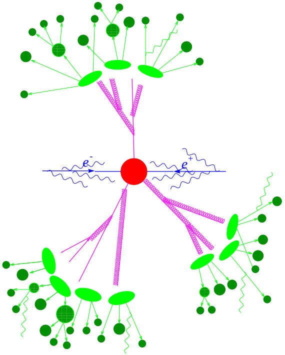
Neutrino ’79: Event 13177 makes history

Image credit: DESY, P. Duinker

The gluon changes everything

[Marchesini,Webber] Nucl.Phys.B238(1984)1, [Webber] Nucl.Phys.B238(1984)492
[Andersson,Gustafson,Ingelman,Sjöstrand] Phys.Rept.97(1983)31

- Short distance interactions
  - Signal process
  - QCD radiative corrections
- Long-distance interactions
  - Hadronization
  - Particle decays



Divide and Conquer

- Quantity of interest: Interaction rate
- Convolution of short & long distance physics

$$\sigma_{ee\to h+X} = \sum_{i\in\{q,g\}} \int \mathrm{d}x \, \underbrace{\hat{\sigma}_{ee\to i+X}(x,\mu_F^2)}_{\text{short distance}} \, \underbrace{D_i^{(h)}(x,\mu_F^2)}_{\text{long distance}}$$

Fourty years and many discoveries later ...

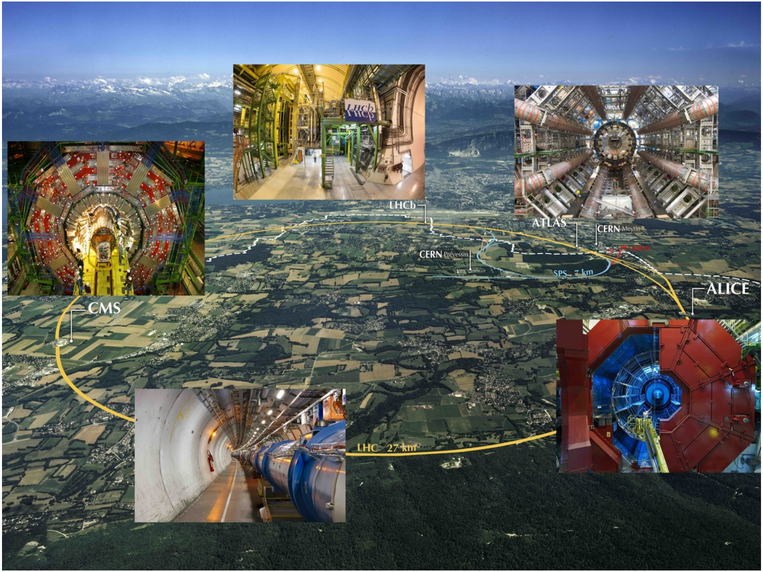
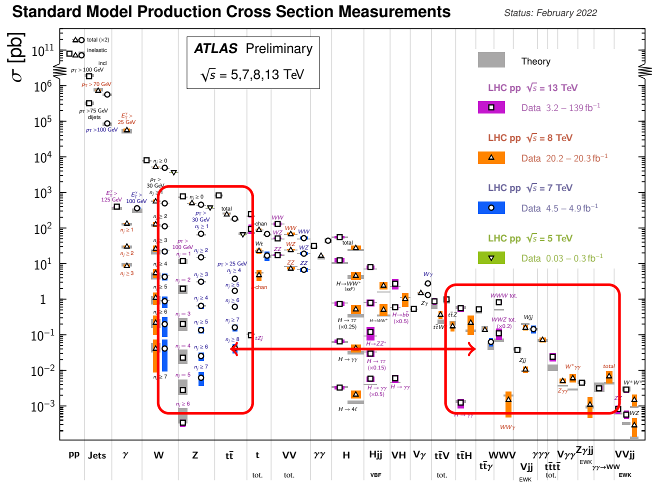


Image credit: CERN

... it’s all about jets



[ATLAS] https://twiki.cern.ch/twiki/bin/view/AtlasPublic/StandardModelPublicResults

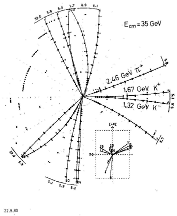
- Signals: High multiplicity but comparably low complexity
- Main backgrounds: High multiplicity and high complexity

So we need to simulate jets ...

Accuracy → Precision

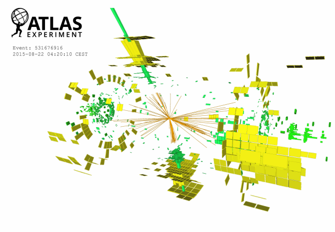
- 1985: Color coherent evolution; Backward evolution
- 1990: Color dipole formalism; Spin correlations
- 1995: Color dipole showers
- 2000: LO merging; NLO matching
- 2005: Dipole-like showers; Antenna showers; NLO merging
- 2010: Color ME corrections; NNLO matching
- 2015: Threshold effects; NLO splitting kernels; Higher-order PS framework
- 2018: Accuracy quantification; Amplitude evolution
- 2022: NLL & NGLs in one framework; Efficient $1/N_c$ corrections; NNLL soft-only evolution

1985 1990 1995 2000 2005 2010 2015 2018 2022

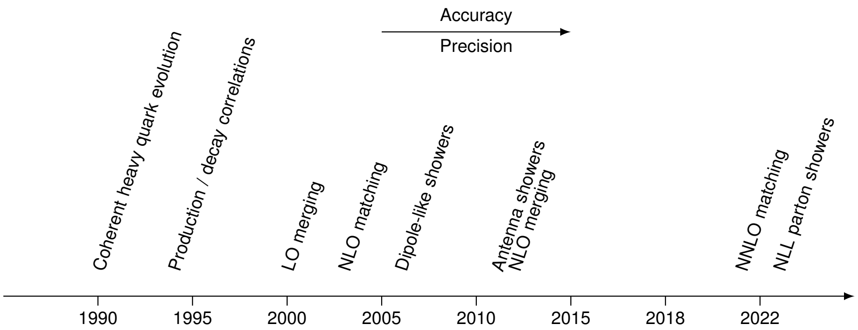


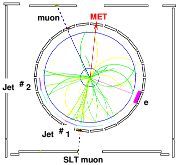
$\sqrt{s} \times 500$ →

e^+e^- vs. pp



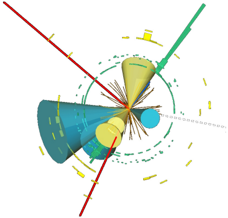
... sometimes fat jets ...





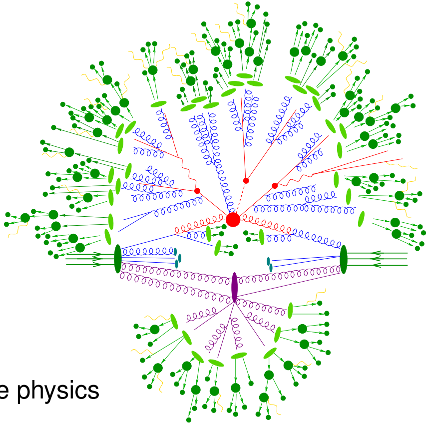
$\sqrt{s} \times 7$

$p\bar{p}$ vs. pp



... but always many jets

[Buckley et al.] arXiv:1101.2599
[Campbell et al.] arXiv:2203.11110



- Short distance interactions
  - Signal process
  - Radiative corrections
- Long-distance interactions
  - Hadronization
  - Particle decays

Divide and Conquer

- Quantity of interest: Interaction rate
- Convolution of short & long distance physics

$$\sigma_{p_1p_2\to X} = \sum_{i,j\in\{q,g\}} \int \mathrm{d}x_1\mathrm{d}x_2\, \underbrace{f_{p_1,i}(x_1,\mu_F^2)f_{p_2,j}(x_2,\mu_F^2)}_{\text{long distance}}\, \underbrace{\hat{\sigma}_{ij\to X}(x_1x_2,\mu_F^2)}_{\text{short distance}}$$

The connection to pQCD theory

- $\hat{\sigma}_{ij\to n}(\mu_F^2) \to$ Collinearly factorized fixed-order result at N^{x}LO

  Implemented in fully differential form to be maximally useful

  Tree level: $\mathrm{d}\Phi_n\; B_n$

  - Automated ME generators + phase-space integrators

  1-Loop level: $\mathrm{d}\Phi_n \left(B_n + V_n + \sum C + \sum I_n\right) + \mathrm{d}\Phi_{n+1}\left(R_n - \sum S_n\right)$

  - Automated loop ME generators + integral libraries + IR subtraction

  2-Loop level: It depends ...

  - Individual solutions based on SCET, q_T subtraction, P2B

- $f_i(x,\mu_F^2) \to$ Collinearly factorized PDF at N^{y}LO

  Evaluated at $O(1\mathrm{GeV}^2)$ and expanded into a series above $1\mathrm{GeV}^2$

  DGLAP: $\displaystyle \frac{\mathrm{d}x\, x f_a(x,t)}{\mathrm{d}\ln t} = \sum_{b=q,g}\int_0^1 \mathrm{d}\tau \int_0^1 \mathrm{d}z\, \frac{\alpha_s}{2\pi}\left[zP_{ab}(z)\right]_+ \tau f_b(\tau,t)\,\delta(x-\tau z)$

  - Parton showers, dipole showers, antenna showers, ...

  Matching: $\displaystyle \mathrm{d}\Phi_n\, \frac{S_n}{B_n} \leftrightarrow \frac{\mathrm{d}t}{t}\mathrm{d}z\, \frac{\alpha_s}{2\pi}\, P_{ab}(z)$

  - MC@NLO, POWHEG, Geneva, MINNLO$_{\mathrm{PS}}$, ...

Simulation of QCD dipole radiation

Approaches, problems & solutions

Semi-classical radiation pattern

[Marchesini,Webber] NPB310(1988)461

- Soft gluon radiator can be written in terms of energies and angles

$$J_\mu J^\mu \to \frac{p_i p_k}{(p_i p_j)(p_j p_k)} = \frac{W_{ik,j}}{E_j^2}$$

Angular “radiator” function

$$W_{ik,j} = \frac{1-\cos\theta_{ik}}{(1-\cos\theta_{ij})(1-\cos\theta_{jk})}$$

- Divergent as $\theta_{ij} \to 0$ and as $\theta_{jk} \to 0$

$\to$ Expose individual collinear singularities using $W_{ik,j} = \tilde{W}^i_{ik,j} + \tilde{W}^k_{ki,j}$

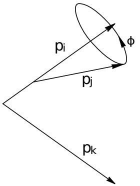
$$\tilde{W}^i_{ik,j} = \frac{1}{2}\left[\frac{1-\cos\theta_{ik}}{(1-\cos\theta_{ij})(1-\cos\theta_{kj})} + \frac{1}{1-\cos\theta_{ij}} - \frac{1}{1-\cos\theta_{kj}}\right]$$

  - Divergent as $\theta_{ij} \to 0$, but regular as $\theta_{kj} \to 0$
  - Convenient properties upon integration over azimuthal angle

Semi-classical radiation pattern

- Work in a frame where direction of $\vec{p}_i$ aligned with z-axis

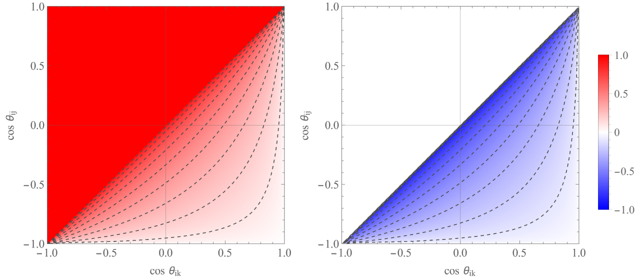
$$\cos\theta_{kj} = \cos\theta_k^i \cos\theta_j^i + \sin\theta_k^i \sin\theta_j^i \cos\phi_{kj}^i$$



- Integration over ϕ_j yields

$$\frac{1}{2\pi}\int_0^{2\pi} \mathrm{d}\phi_{kj}^i \tilde{W}_{ik,j}^i = \frac{1}{1-\cos\theta_j^i} \times \begin{cases} 1 & \text{if} \quad \theta_j^i < \theta_k^i \\ 0 & \text{else} \end{cases}$$

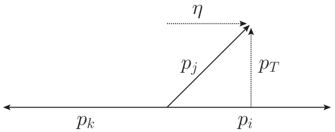
  - On average, no radiation outside cone defined by parent dipole
  - Differential radiation pattern more intricate:
    Positive & negative contributions outside cone sum to zero



Dual description and the Lund plane

[Gustafson] PLB175(1986)453

- Compute everything in center-of-mass frame of fast partons



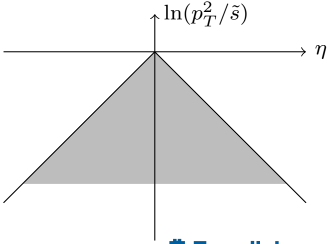
- Simple expressions for transverse momentum and rapidity

$$p_T^2 = \frac{2(p_i p_j)(p_k p_j)}{p_i p_k}\ , \qquad \eta = \frac{1}{2}\ln\frac{p_i p_j}{p_k p_j}$$

- In momentum conserving parton branching $(\tilde{p}_i, \tilde{p}_k) \to (p_i, p_k, p_j)$

$$-\ln \tilde{s}_{ik}/p_T^2 \le 2\eta \le \ln \tilde{s}_{ik}/p_T^2$$

- Differential phase-space element $\propto \mathrm{d}p_T^2\,\mathrm{d}\eta$
- Visualized in Lund plane
  - Phase space bounded by diagonals
  - Single-emission semi-classical radiation probability a constant



Angular ordered parton showers

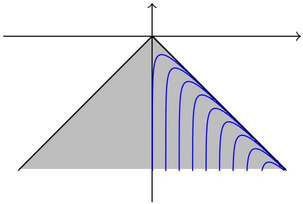
[Marchesini,Webber] NPB238(1984)1, . . .

- Differential radiation probability

$$\mathrm{d}\mathcal{P} = \mathrm{d}\Phi_{+1}|M|^2 \approx \frac{\mathrm{d}\tilde{q}^2}{\tilde{q}^2}\,\mathrm{d}z\,\frac{\alpha_s}{2\pi}\,P_{\widetilde{ij}i}(z)$$

  - Ordering parameter $\tilde{q}^2 = \frac{2p_ip_j}{z(1-z)} \approx 4E_{\widetilde{ij}}^2\sin^2\frac{\theta_{ij}}{2}$
  - Splitting variable $z = \frac{1+\cos\theta_{ik}}{2} = \frac{p_ip_k}{(p_i+p_j)p_k}$

- Lund plane filled from center to edges
  - Random walk in p_T^2
  - Color factors correct for observables insensitive to azimuthal correlations
  - Small dead zone at $\ln(p_T^2/\tilde{s}) \approx 0$



- Usually disfavored due to dead zones
  Not suitable to resum non-global logartihms

Dipole showers

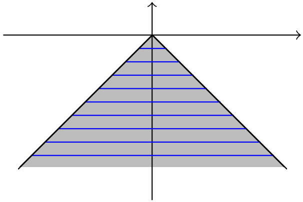
[Gustafson,Pettersson] NPB306(1988)746, ...

- Differential radiation probability for the dipole

$$\mathrm{d}\mathcal{P} = \mathrm{d}\Phi_{+1}|M|^2 \approx \frac{\mathrm{d}p_T^2}{p_T^2}\,\mathrm{d}\eta\,\frac{\alpha_s}{2\pi}\,\tilde{P}_{\widetilde{ij}}(z)$$

  - Ordering parameter p_T^2
  - Splitting variable $z = 1 - \frac{s_{ij}}{s - s_{ij}}\,\mathrm{e}^{-2\eta}$

- Lund plane filled from top to bottom
  - Random walk in η
  - Color factors in CFFE approximation
  - Pairs of partons evolve simultaneously
  - No dead zones

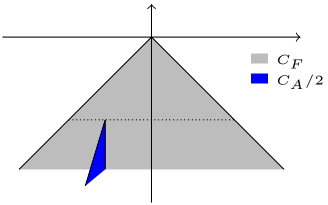


- Solves problem of dead zones
  Known issues with color coherence

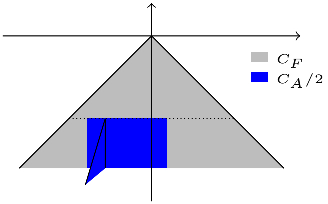
Problems with average color charges

[Gustafsson] NPB392(1993)251

- In angular ordered showers angles are measured in the event center-of-mass frame
  $\rightarrow$ coherence effects modeled by angular ordering variable agree on average with matrix element



- In dipole-like showers angles effectively measured in center-of-mass frame of emitting color dipole
  $\rightarrow$ angular coherence not reflected by setting average QCD charge

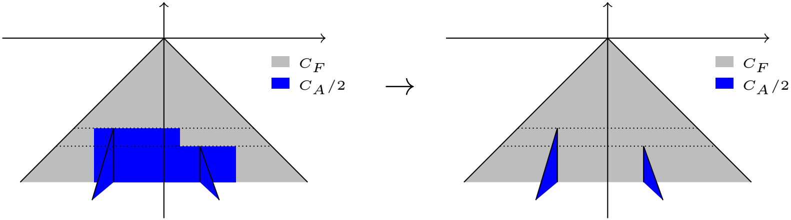


- Emission off "back plane" in Lund diagram should be associated with C_F, but is partly associated with $C_A/2$ in dipole showers
- All-orders problem that appears first in 2-gluon emission case

Solutions for average color charges

[Gustafsson] NPB392(1993)251
[Hamilton,Medves,Salam,Scyboz,Soyez] arXiv:2011.10054

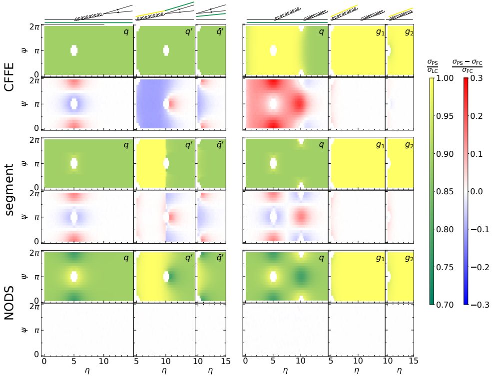
- Analyze rapidity of gluon emission in event center-of-mass frame
- Sectorize phase space and assign gluon to closest parton
  $\rightarrow$ choose corresponding color charge for evolution
- Same technology for higher number of emissions



- Starting with 4 emissions, there be "color monsters"
  [Dokshitzer,Troian,Khoze] SJNP47(1988)881, YF47(1988)1384
  - Quartic Casimir operators (easy)
  - Non-factorizable contributions (hard)

Solutions for average color charges

[Hamilton,Medves,Salam,Scyboz,Soyez] arXiv:2011.10054



- Can include double-soft corrections via reweighting [Giele,Kosower,Skands] arXiv:1102.2126
- Algorithm scales as N^2 but can be simplified while retaining formal accuracy
- Implementation as nested corrections in rapidity segments of parent dipole
- Excellent agreement with full matrix element
- Good agreement with full-color evolution [Hatta,Ueda] arXiv:1304.6930

Problems with momentum mapping

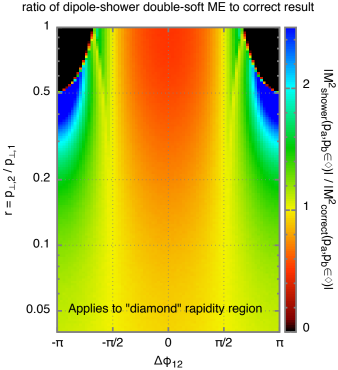
[Dasgupta,Dreyer,Hamilton,Monni,Salam] arXiv:1805.09327

- Subtle problems in standard dipole-like momentum mapping

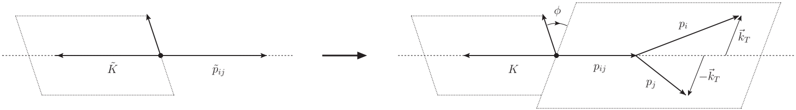
$$p_k^\mu = \left(1 - \frac{p_{ij}^2}{2\tilde{p}_{ij}\tilde{p}_k}\right)\tilde{p}_k^\mu$$

$$p_i^\mu = \tilde{z}\,\tilde{p}_{ij}^\mu + (1-\tilde{z})\frac{p_{ij}^2}{2\tilde{p}_{ij}\tilde{p}_k}\tilde{p}_k^\mu + k_\perp^\mu$$

$$p_j^\mu = (1-\tilde{z})\,\tilde{p}_{ij}^\mu + \tilde{z}\frac{p_{ij}^2}{2\tilde{p}_{ij}\tilde{p}_k}\tilde{p}_k^\mu - k_\perp^\mu$$



- Induces angular correlations across multiple emissions
- Spoils agreement w/ analytic resummation



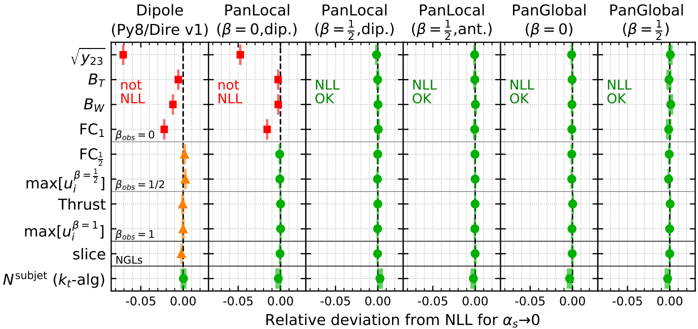
Solutions for momentum mapping

[Dasgupta,Dreyer,Hamilton,Monni,Salam,Soyez] arXiv:2002.11114

- Problem can be solved by partitioning of antenna radiation pattern and choosing a suitable evolution variable ($\beta \sim 1/2$)

$$k_T = \rho v e^{\beta|\bar{\eta}|} \qquad \rho = \left(\frac{s_i s_j}{Q^2 s_{ij}}\right)^{\beta/2}$$

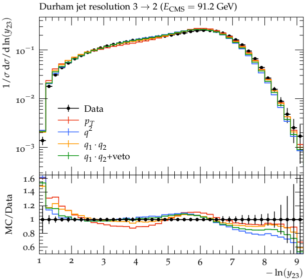
- Different recoil schemes can lead to NLL result if β chosen appropriately: Local dipole, local antenna, and global antenna
- NLL correct for global and non-global observables in $e^+e^- \to$hadrons

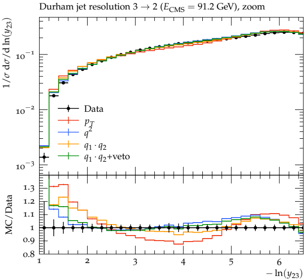


Solutions for momentum mapping

[Bewick,Ferrario-Ravasio,Richardson,Seymour] arXiv:1904.11866

- Recoil schemes affect logarithmic accuracy but impact also phase-space coverage
- In context of angular ordered Herwig 7 (NLL accurate for global observables)
    - q_T preserving scheme:
      Maintains logarithmic accuracy
      Overpopulates hard region
    - q^2 preserving scheme:
      Breaks logarithmic accuracy
      Good description of hard region
    - Dot product preserving scheme (new):
      Maintains logarithmic accuracy
      Good description of hard radiation

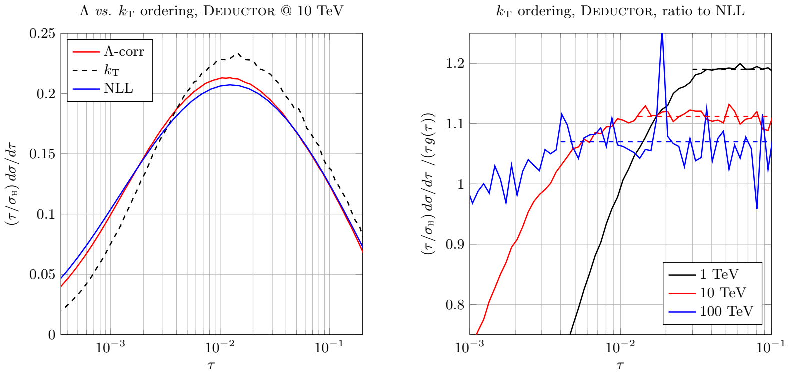




Solutions for momentum mapping

[Nagy,Soper] arXiv:2011.04773

- Local transverse recoil, global longitudinal recoil
- Analytic proof of NLL correctness, based on kinematics in $s \to \infty$ limit



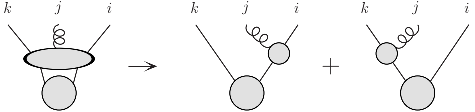
A new perspective on old ideas

Identified partons & azimuthal angle dependence

The semi-classical matrix element revisited

- Alternative to additive matching: partial fraction matrix element & match to collinear sectors [Ellis,Ross,Terrano] NPB178(1981)421, [Catani,Seymour] hep-ph/9605323

$$\frac{W_{ik,j}}{E_j^2} \rightarrow \frac{1}{p_i p_j}\frac{p_i p_k}{(p_i+p_k)p_j} + \frac{1}{p_k p_j}\frac{p_i p_k}{(p_i+p_k)p_j}$$



  - Captures matrix element both in angular ordered and unordered region
  - Caveat: Oversampling difficult for certain kinematics maps

- Separate into energy & angle first [Herren,Krauss,Reichelt,Schönherr,SH] arXiv:2208.06057
  Partial fraction angular radiator only: $W_{ik,j} = \bar{W}^i_{ik,j} + \bar{W}^k_{ki,j}$

$$\bar{W}^i_{ik,j} = \frac{1-\cos\theta_{ik}}{(1-\cos\theta_{ij})(2-\cos\theta_{ij}-\cos\theta_{kj})}$$

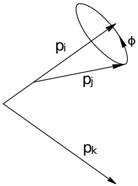
  - Bounded by $(1-\cos\theta_{ij})\bar{W}^i_{ik,j} < 2$
  - Strictly positive

The semi-classical matrix element revisited

- Integration over ϕ_j yields

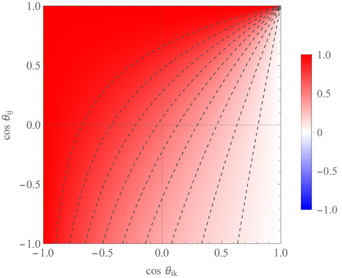
$$\frac{1}{2\pi}\int_0^{2\pi} \mathrm{d}\phi^i_{kj}\,\bar{W}^i_{ik,j} = \frac{1}{1-\cos\theta^i_j}\,\frac{1}{\sqrt{(\bar{A}^i_{ij,k})^2-(\bar{B}^i_{ij,k})^2}}$$

  - Radiation across all of phase space
  - Probabilistic radiation pattern

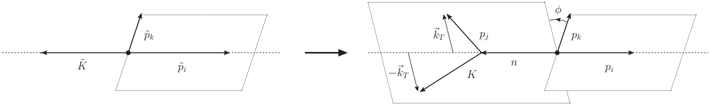


$$\bar{A}^i_{ij,k} = \frac{2-\cos\theta^i_j(1+\cos\theta^i_k)}{1-\cos\theta^i_k}$$

$$\bar{B}^i_{ij,k} = \frac{\sqrt{(1-\cos^2\theta^i_j)(1-\cos^2\theta^i_k)}}{1-\cos\theta^i_k}$$



Kinematics mapping revisited



- In collinear limit, splitting kinematics defined by ($n \to$ auxiliary vector)

$$p_i \xrightarrow{i||j} z\,\tilde{p}_i\;, \qquad p_j \xrightarrow{i||j} (1-z)\,\tilde{p}_i \qquad \text{where} \qquad z = \frac{p_i n}{(p_i+p_j)n}$$

- Parametrization, using hard momentum $\tilde{K}$

$$p_i = z\,\tilde{p}_i\;, \qquad n = \tilde{K} + (1-z)\,\tilde{p}_i$$

- Using on-shell conditions & momentum conservation ($\kappa = \tilde{K}^2/(2\tilde{p}_i\tilde{K})$)

$$\begin{aligned} p_j &= (1-z)\,\tilde{p}_i + v\big(\tilde{K} - (1-z+2\kappa)\,\tilde{p}_i\big) + k_\perp \\ K &= \tilde{K} - v\big(\tilde{K} - (1-z+2\kappa)\,\tilde{p}_i\big) - k_\perp \end{aligned}$$

- Momenta in $\tilde{K}$ Lorentz-boosted to new frame K [Catani,Seymour] hep-ph/9605323

$$p_l^\mu \to \Lambda^\mu{}_\nu(K,\tilde{K})\,p_l^\nu\;, \qquad \Lambda^\mu{}_\nu(K,\tilde{K}) = g^\mu{}_\nu - \frac{2(K+\tilde{K})^\mu(K+\tilde{K})_\nu}{(K+\tilde{K})^2} + \frac{2\tilde{K}^\mu K_\nu}{K^2}\;.$$

Logarithmic accuracy – Analytic proof

- Logarithmic accuracy of parton shower can be quantified by comparing results to (semi-)analytic resummation e.g. [Banfi,Salam,Zanderighi] hep-ph/0407286
- Example: Thrust or FC_0 in $e^+e^- \rightarrow$hadrons

- Define a shower evolution variable $\xi = k_T^2/(1-z)$
- Parton-shower one-emission probability for $\xi > Q^2\tau$

$$R_{\mathrm{PS}}(\tau) = 2\int_{Q^2\tau}^{Q^2} \frac{d\xi}{\xi} \int_{z_{\min}}^{z_{\max}} dz \, \frac{\alpha_s\left(k_T^2\right)}{2\pi} \, C_F \left[\frac{2}{1-z} - (1+z)\right] \Theta(\eta)$$

- Approximate to NLL accuracy

$$R_{\mathrm{NLL}}(\tau) = 2\int_{Q^2\tau}^{Q^2} \frac{d\xi}{\xi} \left[\int_0^1 dz \, \frac{\alpha_s\left(k_T^2\right)}{2\pi} \frac{2\,C_F}{1-z} \Theta(\eta) - \frac{\alpha_s(\xi)}{\pi} \, C_F B_q\right]$$

Logarithmic accuracy – Analytic proof

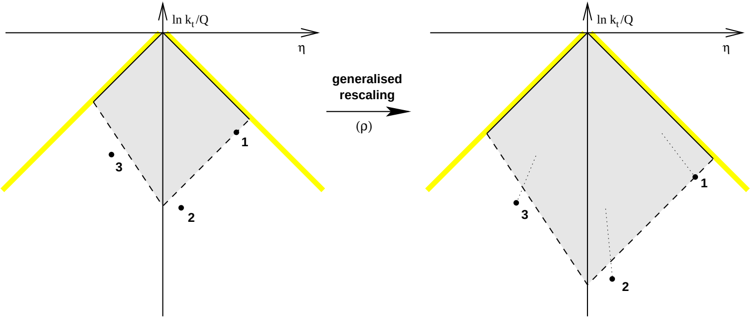
- Cumulative cross section $\Sigma\,(\tau) = e^{-R(\tau)}\mathcal{F}\,(\tau)$ obtained from all-orders resummed result by Taylor expansion of virtual corrections in cutoff ε

$$\mathcal{F}(\tau) = \int \mathrm{d}^3k_1|M(k_1)|^2\, e^{-R'\,\ln\frac{\tau}{\varepsilon v_1}} \sum_{m=0}^{\infty}\frac{1}{m!}\left(\prod_{i=2}^{m+1}\int_{\varepsilon v_1}^{v_1}\mathrm{d}^3k_i|M(k_i)|^2\right) \\ \times\,\Theta\big(\tau - V(\{p\},k_1,\ldots,k_n)\big)$$

  - $\mathcal{F}(\tau)$ is pure NLL & accounts for (correlated) multiple-emission effects
- In order to make $\mathcal{F}(\tau)$ calculable, make the following assumptions
  - Observable is recursively infrared and collinear safe
  - Hold $\alpha_s(Q^2)\ln\tau$ fixed, while taking limit $\tau\to 0$
    $\to$ Can factorize integrals and neglect kinematic edge effects

    Can be interpreted as $\alpha_s\to 0$ or $s\to\infty$ limit

Logarithmic accuracy – Analytic proof



- $\alpha_s \to 0$ / $s \to \infty$ limit taken by similarity transformation of Lund plane
- Can be parametrized in terms of scaling parameter ρ

$$k_{t,l} \to k'_{t,l} = k_{t,l}\,\rho^{(1-\xi_l)/a+\xi_l/(a+b)}$$

$$\eta_l \to \eta'_l = \eta - \xi_l \frac{\ln\rho}{a+b}\;, \qquad \text{where} \qquad \xi = \frac{\eta}{\eta_{\max}}$$

observable parametrization at one-emission level: $v = (k_t^2/Q^2)^a \exp(-b\eta)$

- NLL precision requires scaling to be maintained after additional emissions

Logarithmic accuracy – Analytic proof

- Lorentz transformation defined by shift $\tilde{K} \to K$

$$K^\mu = \tilde{K}^\mu - X^\mu\ , \qquad \text{where} \qquad X^\mu = p_j^\mu - (1-z)\,\tilde{p}_i^\mu$$

- X is small, but is it small enough? Rewrite

$$\Lambda^\mu_{\ \nu}(K,\tilde{K}) = g^\mu_\nu + \tilde{K}^\mu A_\nu + X^\mu B_\nu$$

- In NLL limit, coefficients scale as

$$A^\nu \xrightarrow{\rho\to 0} 2\,\frac{\tilde{K}X}{\tilde{K}^2}\,\frac{\tilde{K}^\nu}{\tilde{K}^2} - \frac{X^\nu}{\tilde{K}^2}\ , \qquad \text{and} \qquad B^\nu \xrightarrow{\rho\to 0} \frac{\tilde{K}^\nu}{\tilde{K}^2}\ .$$

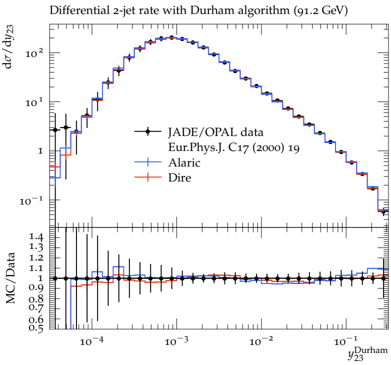
- Simplify situation by taking $a=1$, $b=0$ (worst offenders)
  Relative momentum shift of soft emission particle l becomes

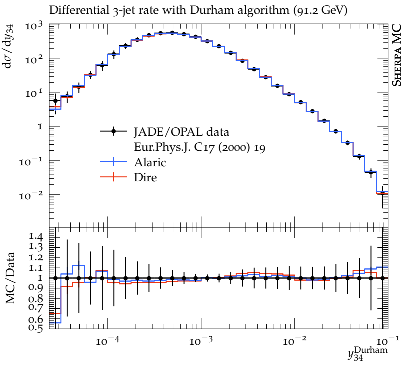
$$\Delta p_l^{0,3}/\tilde{p}_l^{0,3} \sim \rho^{1-\max(\xi_i,\xi_j)} \quad \xrightarrow{\rho\to 0} \quad 0$$
$$\Delta p_l^{1,2}/\tilde{p}_l^{1,2} \sim \rho^{1-\xi_l} \quad \xrightarrow{\rho\to 0} \quad 0$$

- For hard momenta, leading terms in X^μ cancel exactly
  Remaining components scale as ρ or stronger

$e^+e^- \rightarrow$ hadrons

[Herren,Krauss,Reichelt,Schönherr,SH] arXiv:2208.06057

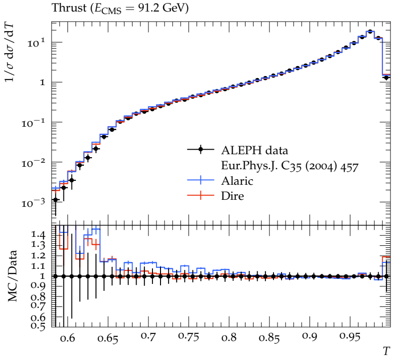


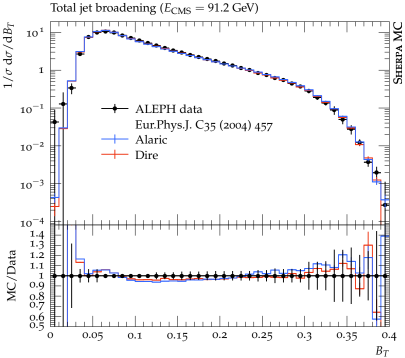


- Comparison to experimental data from LEP

$e^+e^- \to$hadrons

[Herren,Krauss,Reichelt,Schönherr,SH] arXiv:2208.06057

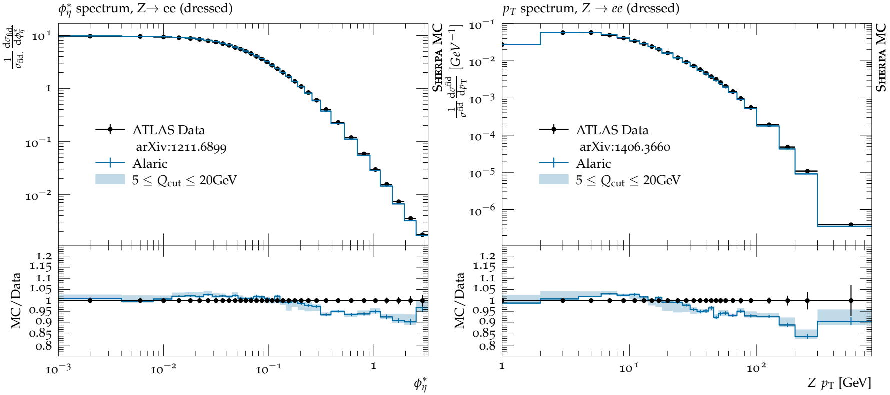




- Comparison to experimental data from LEP

Drell-Yan lepton pair production

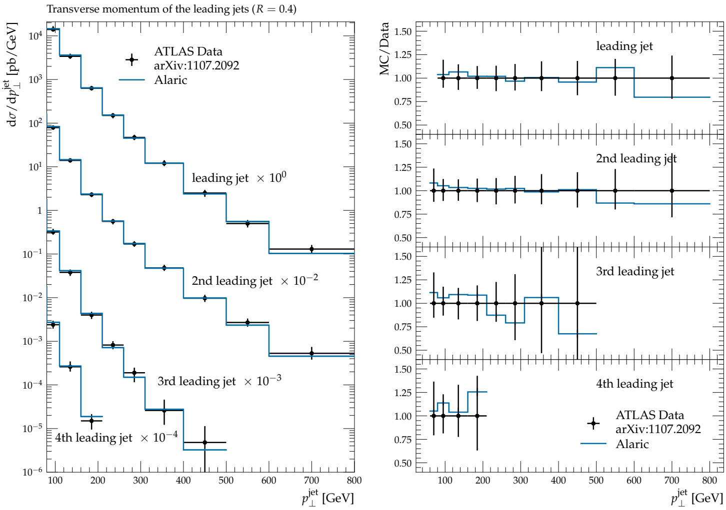
[Krauss,Reichelt,SH] arXiv:2404.14360



- Comparison to experimental data from LHC
- Leading-order multi-jet merging with up to two jets

Jet production

[Krauss,Reichelt,SH] arXiv:2404.14360



- Comparison to experimental data from LHC, parton shower only

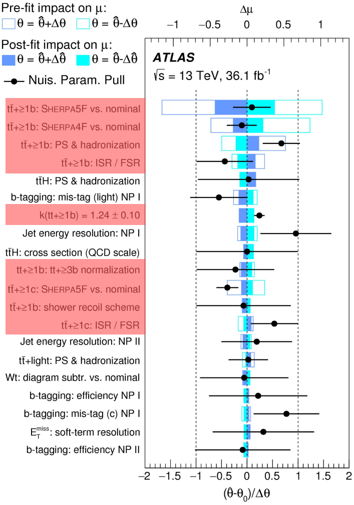
Heavy quark effects in parton showers

A fresh perspective

Experimental observations

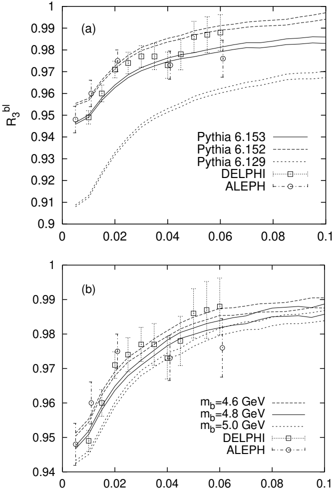
- Example $t\bar{t}b\bar{b}$: MC single largest source of uncertainty on signal strength
- Despite intense study of HF production
  - Fixed order, NLL, FONLL
    [Cacciari,Frixione,Houdeau,Mangano,Nason,Ridolfi,. . .] arXiv:1205.6344, hep-ph/0312132, hep-ph/9801375, NPB373(1992)295
  - In context of particle-level Monte Carlo
    [Marchesini,Webber] NPB330(1990)261,
    [Norrbin,Sjöstrand] hep-ph/0010012,
    [Gieseke,Stephens,Webber] hep-ph/0310083,
    [Schumann,Krauss] arXiv:0709.1027,
    [Gehrmann-deRidder,Ritzmann,Skands] arXiv:1108.6172,
    [Assi,SH] arXiv:2307.00728
- Recurring themes, not special to $t\bar{t}b\bar{b}$
  - PS uncertainties hard to judge and reduce
    [Cascioli,Maierhöfer,Moretti,Pozzorini,Siegert] arXiv:1309.591
  - Matching needed for inclusive predictions
    [Krause,Siegert,SH] arXiv:1904.09382,
    [Ferencz,Katzy,Krause,Pollard,Siegert,SH]

[ATLAS] arXiv:1712.08895



Theory problems

- Both high-energy limit and threshold region of heavy-flavor production to be modeled, but a number of obstacles:
- Infrared finite prediction for $g \to Q\bar{Q}$ leaves splitting functions somewhat arbitrary
- Soft gluon emission off light/heavy quarks associated with $\alpha_s(k_T^2)$, i.e. "correct" scale is k_T^2 [Amati et al.] NPB173(1980)429, but no such argument to set scale for $g \to Q\bar{Q}$
  $\to$ HQ production rate not very stable w.r.t. parton shower variations
- A number of different prescriptions, e.g. [Marchesini,Webber] NPB330(1990)261, [Norrbin,Sjöstrand] hep-ph/0010012, [Gieseke,Stephens,Webber] hep-ph/0310083, [Schumann,Krauss] arXiv:0709.1027, [Gehrmann-deRidder,Ritzmann,Skands] arXiv:1108.6172, [Assi,SH] arXiv:2307.00728



[Norrbin,Sjöstrand] hep-ph/0010021

Soft-collinear matching for heavy quarks

[Marchesini,Webber] NPB330(1990)261

- Singularity in angular radiator screened by velocity $\rightarrow$ deadcone $\theta_0 \approx m/E$

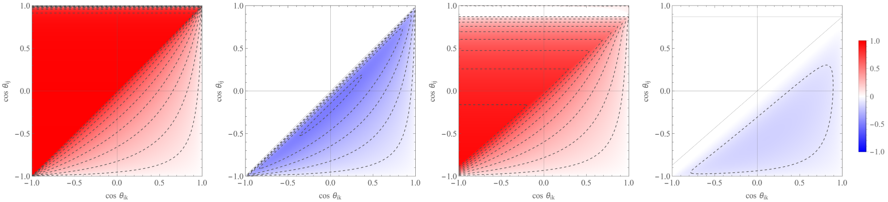
$$W_{ik,j} = \frac{1 - v_i v_k \cos\theta_{ik}}{(1 - v_i \cos\theta_{ij})(1 - v_k \cos\theta_{jk})} - \frac{(1 - v_i^2)/2}{(1 - v_i \cos\theta_{ij})^2} - \frac{(1 - v_k^2)/2}{(1 - v_k \cos\theta_{jk})^2}$$

- Quasi-collinear divergence if $m_Q \propto k_T$ as $k_T \rightarrow 0$
  $\rightarrow$ Expose individual singularities via $W_{ik,j} = \tilde{W}^i_{ik,j} + \tilde{W}^k_{ki,j}$

$$\tilde{W}^i_{ik,j} = \frac{1}{2(1 - v_i \cos\theta_{ij})}\left[\left(\frac{1 - v_i v_k \cos\theta_{ik}}{1 - v_k \cos\theta_{kj}} - \frac{1 - v_i^2}{1 - v_i \cos\theta_{ij}}\right) + 1 - \frac{1 - v_i \cos\theta_{ij}}{1 - v_k \cos\theta_{kj}}\right]$$

- Approximate angular ordering after azimuthal averaging

$v^2 = 1 - m_b^2/m_Z^2$ $\qquad$ $v^2 = 1 - m_t^2/(350\ \mathrm{GeV})^2$



A novel approach to heavy-quark evolution

[Assi,SH] arXiv:2307.00728

- Alternative approach: separate into energy & angle first
  Partial fraction angular radiator only: $W_{ik,j} = \bar{W}^i_{ik,j} + \bar{W}^k_{ki,j}$

$$\bar{W}^i_{ik,j} = \frac{1 - v_k \cos\theta_{kj}}{2 - v_i \cos\theta_{ij} - v_k \cos\theta_{kj}}\, W_{ik,j}$$

- Can be written in more intuitive form (n^μ defines reference frame)

$$\bar{W}^i_{ik,j} = \frac{1}{2 l_i l_j}\left(\frac{l_{ik}^2}{l_{ik} l_j} - \frac{l_i^2}{l_i l_j} - \frac{l_k^2}{l_k l_j}\right)\,, \qquad \text{where} \qquad l_i^\mu = \sqrt{n^2}\,\frac{p_i^\mu}{p_i n}$$

- Quasi-collinear limit manifest

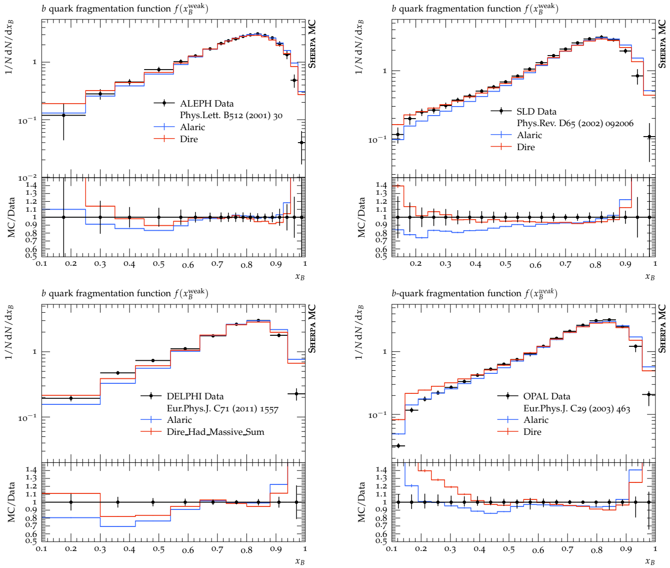
$$\frac{\bar{W}_{ik,j}}{E_j^2} \xrightarrow[m_i \propto p_i p_j]{i||j} w^{(\mathrm{coll})}_{ik,j}(z) := \frac{1}{2 p_i p_j}\left(\frac{2z}{1-z} - \frac{m_i^2}{p_i p_j}\right)$$

- Matching to massive DGLAP splitting functions

$$\frac{P_{(ij)i}(z,\varepsilon)}{(p_i+p_j)^2 - m_{ij}^2} \to \frac{P_{(ij)i}(z,\varepsilon)}{(p_i+p_j)^2 - m_{ij}^2} + \delta_{(ij)i}\,\mathbf{T}_i^2\left[\frac{\bar{W}^i_{ik,j}}{E_j^2} - w^{(\mathrm{coll})}_{ik,j}(z)\right]\,,$$

A novel approach to heavy-quark evolution

[Assi,SH] arXiv:2307.00728



A novel approach to heavy quark production

- Two different approaches to dealing with heavy-quark masses:
  - 4-flavor scheme (4FS): Decoupling scheme - (no b-quarks in PDF)
  - 5-flavor scheme (5FS): Minimal subtraction scheme
- Calculations can be matched by
  - Re-expressing both in same renormalization scheme
  - Subtracting the overlap

$$\sigma^{\text{FONLL}} = \sigma^{\text{massive}} + (\sigma^{\text{massless}} - \sigma^{\text{massive},\,0})$$

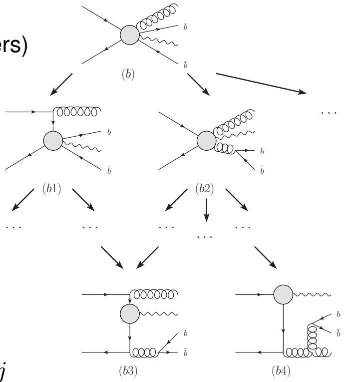
- This has been applied extensively to inclusive observables and is know as fixed-order next-to-leading log (FONLL) scheme

  [Cacciari,Frixione,Mangano,Nason,Ridolfi] hep-ph/0312132, [Forte,Napoletano,Ubiali] arXiv:1508.01529, arXiv:1607.00389, ...
- Extension to differential observables is needed for MC simulations

A novel approach to heavy quark production

[Krause,Siegert,SH] arXiv:1904.09382

- **Interpret $X + b\bar{b}$ as part of $X + jj$**
  1. Cluster to obtain parton shower history
  2. Apply $\alpha_s(\mu_R^2) \to \alpha_s(p_T^2)$ reweighting
  3. Apply Sudakov factors $\Delta(t, t')$ (trial showers)
- **Remove double-counting**
  1. Cluster PS-level event using inverse PS
  2. Look at leading two emissions
     - Heavy Flavour $\to$ keep from $Xb\bar{b}$ ("direct component")
     - Light Flavour $\to$ keep from X+jets ("fragmentation component")
     - Subleading $g \to b\bar{b}$ splittings not from $Xb\bar{b}$ ME, but $X4j$ ME+PS
- **Match 5F$\to$4F in PDFs and α_s**
  1. Use 5F PDF / α_s to be consistent with Xjj
  2. Use matching coefficients to correct to 4F scheme
     [Buza,Matiounine,Smith,van Neerven] hep-ph/9612398, [Forte,Napoletano,Ubiali] arXiv:1607.00389
     $\to$ Coefficients up to (N)LL generated by (N)LO parton shower!
  3. Reweighting needed only for α_s in hard ME

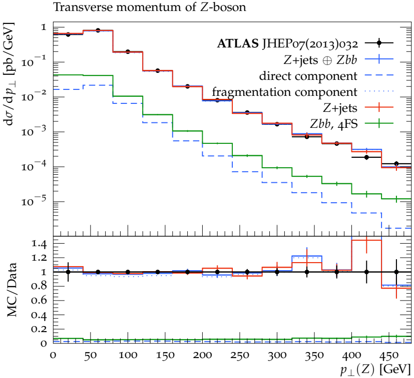


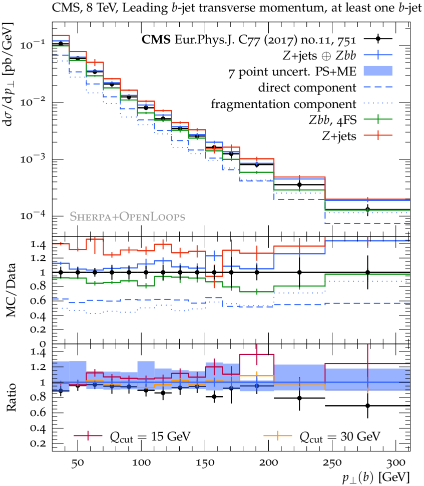
Can be applied to LO and NLO merging!

Application to Z+jets & $Zb\bar{b}$

- Validation with LHC data

[Krause,Siegert,SH] arXiv:1904.09382



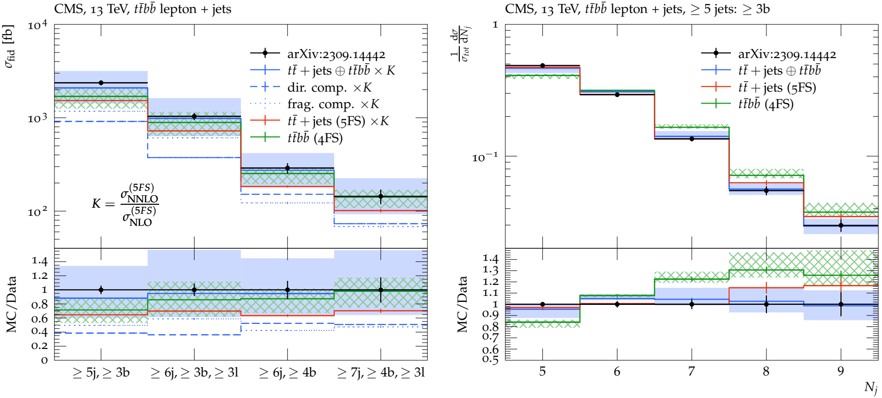


	Data [pb]	**Fusing** [pb]
$Z+\geq 1b$	$3.55 \pm 0.24_{\text{comb}}$	$3.80(5) \pm {}^{0.83}_{0.33}$
$Z+\geq 2b$	$0.331 \pm 0.037_{\text{comb}}$	$0.282(4) \pm {}^{0.027}_{0.022}$

Application to $t\bar{t}$+jets & $t\bar{t}b\bar{b}$

[J. Krause] PhD thesis, [Ferencz,Katzy,Siegert,SH] arXiv:2402.15497

- Combination of $t\bar{t}$+0,1j@NLO+2,3j@LO and massive $t\bar{t}b\bar{b}$@NLO
- 2-bjet production dominated by precise calculation of direct component



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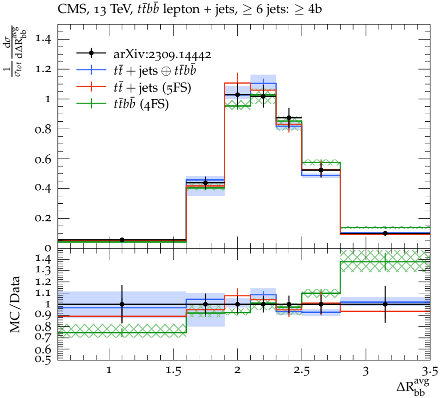
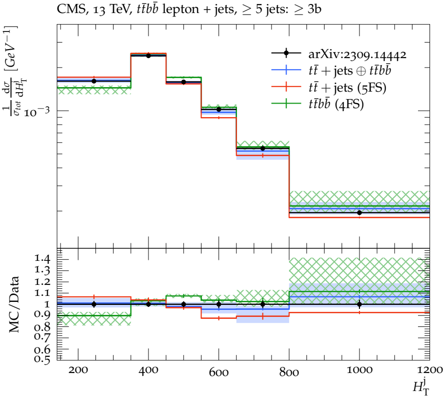
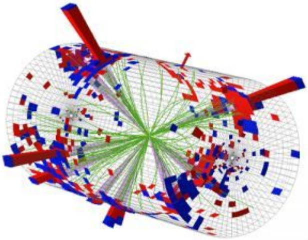
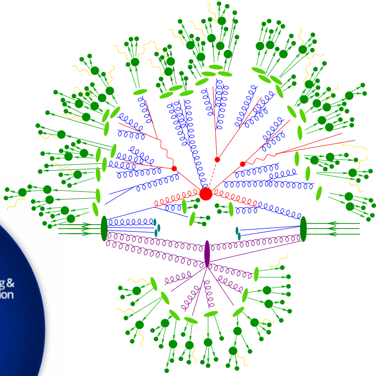
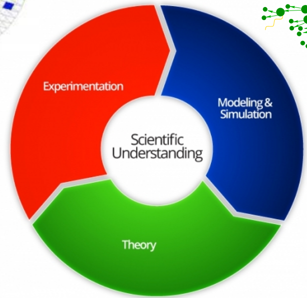




Half a century of teamwork ...







$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i\bar{\psi}\not{D}\psi + h.c.$$

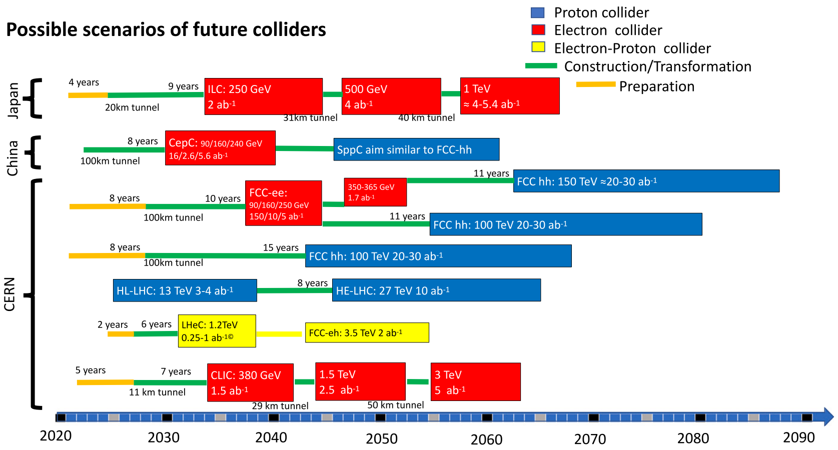
... and we're only getting started

- Fixed-order calculations (not covered here)
  - Higher-order matrix element calculations
  - Higher-order fully differential IR subtraction
  - Computing improvements
- Parton showers
  - Improved logarithmic precision
  - Higher-order splitting kernels
  - Interplay with analytic resummation
- Matching and merging
  - The role of unitarity constraints
  - Interplay with analytic resummation
  - Fully differential higher-order matching

Apologies for only selecting a small subset of topics
For a comprehensive overview: [Campbell et al.] arXiv:2203.11110

Whatever the future may hold ...

[Gray] Rev.Phys. 6 (2021) 100053



... nothing goes without simulations