FERMILAB-SLIDES-25-0111-T

Introduction to Matching and Merging

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MCnet summer school

Stirling, 06/09/2025

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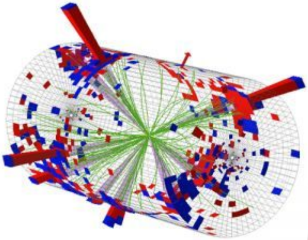
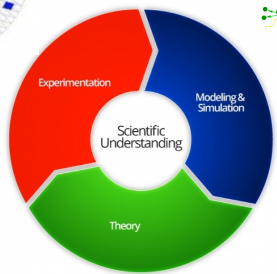
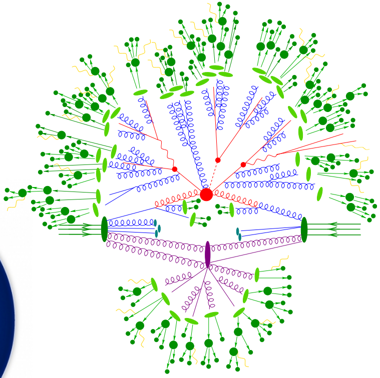
Suggested reading

- R. K. Ellis, W. J. Stirling, B. R. Webber
  QCD and Collider Physics
  Cambridge University Press, 2003
- R. D. Field
  Applications of Perturbative QCD
  Addison-Wesley, 1995
- T. Sjöstrand, S. Mrenna, P. Z. Skands
  PYTHIA 6.4 Physics and Manual
  JHEP 05 (2006) 026
- L. Dixon, F. Petriello (Editors)
  Journeys Through the Precision Frontier
  Proceedings of TASI 2014, World Scientific, 2015

Event generators in the bigger picture





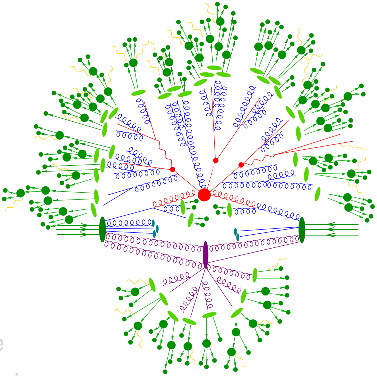


$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i \bar{\Psi} \not{D} \Psi + h.c.$$

Schematics of LHC simulations

Need to cover large dynamic range

- Short distance interactions
  - Signal process
  - Radiative corrections
- Long-distance interactions
  - Hadronization
  - Particle decays



Divide and Conquer

- Quantity of interest: Total interaction rate
- Convolution of short & long distance physics

$$\sigma_{p_1p_2\to X} = \sum_{i,j\in\{q,g\}} \int \mathrm{d}x_1\mathrm{d}x_2 \underbrace{f_{p_1,i}(x_1,\mu_F^2) f_{p_2,j}(x_2,\mu_F^2)}_{\text{long distance}} \underbrace{\hat{\sigma}_{ij\to X}(x_1x_2,\mu_F^2)}_{\text{short distance}}$$

Matching & merging unifies signal process & radiative corrections

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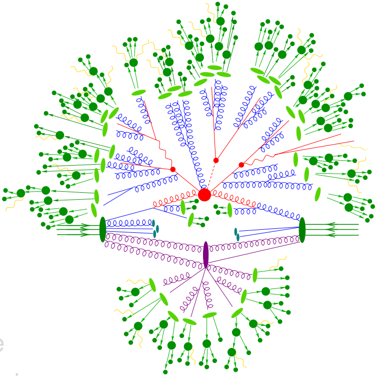

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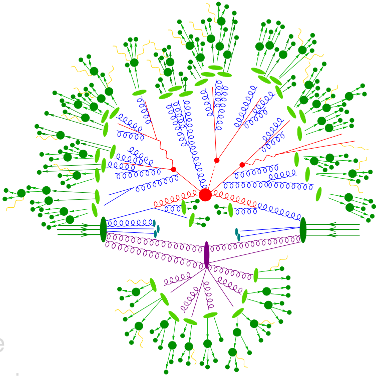

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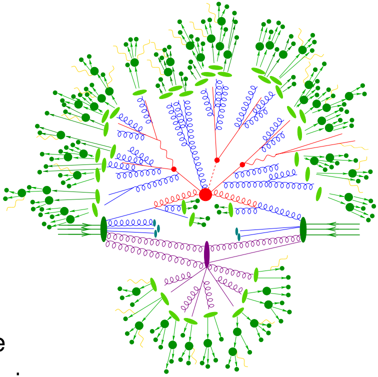

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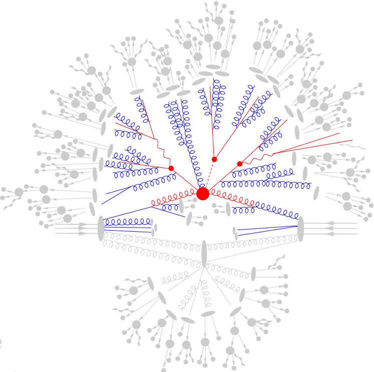

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Matching & merging unifies signal process & radiative corrections

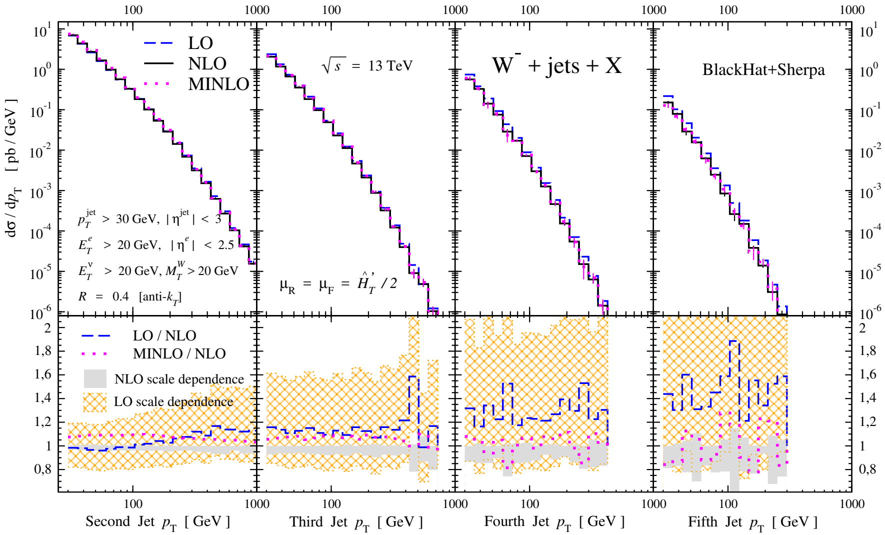
Outline of lectures

- Why match and merge?
  - Problems with fixed-order QCD
  - Problems with parton showers
- Theory background
  - Matching to NLO calculations
  - Merging of (N)LO calculations
  - Fusing of merged results
- Practicalities
  - What's my observable?
  - What does pQCD predict?
  - Common pitfalls

Why match and merge?

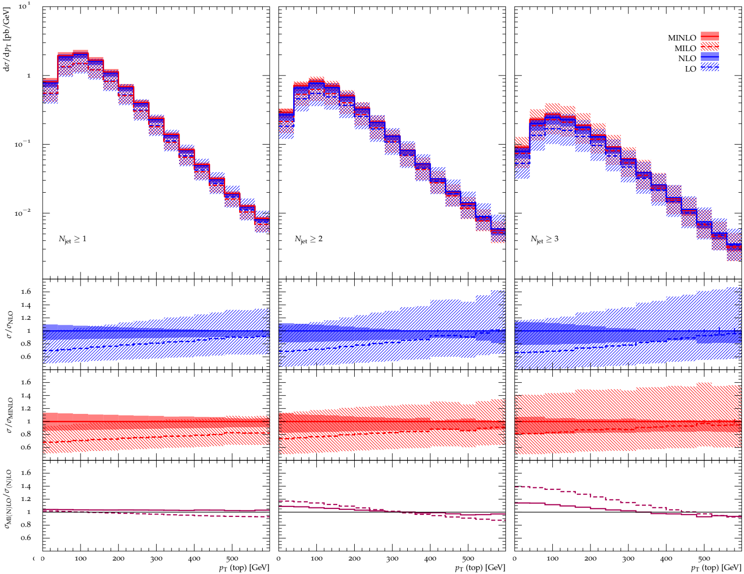
Typical fixed-order pQCD performance: $W^{\pm}$+5 jets

[Bern et al.] arXiv:1304.1253, [Anger at al.] arXiv:1712.08621



Typical fixed-order pQCD performance: $t\bar{t}$+3 jets

[Maierhöfer et al.] arXiv:1607.06934



When fixed-order calculations don't work ...

... one of the assumptions of fixed-order pQCD must be violated

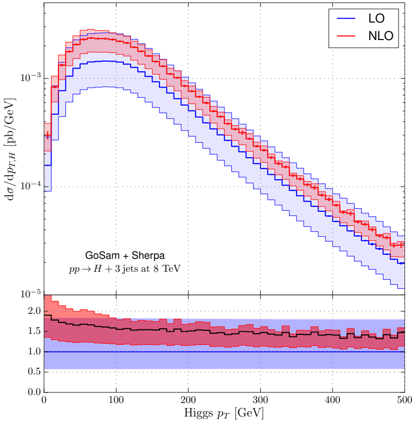
- Scale hierarchies or rapidity gaps become large, leading to logarithmically enhanced corrections
- Flavor content of jets is resolved in some detail such that specifics of fragmentation are relevant
- Flavor channels have been down-selected to simplify computation (e.g. 4-flavor scheme in inclusive region)
- Scales are chosen inappropriately
- Reasons not relevant for this lecture ...

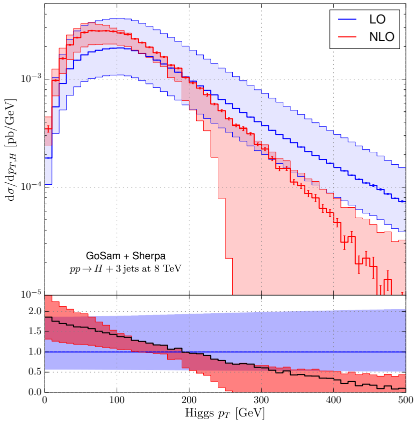
If none of these apply and your fixed-order calculation gives nonsense:

There is a problem with the calculation or the way it's used.
Matching & merging won't solve it.
You should talk to the authors!

Poor performance example: Scale choice

[Greiner et al.] arXiv:1506.01016



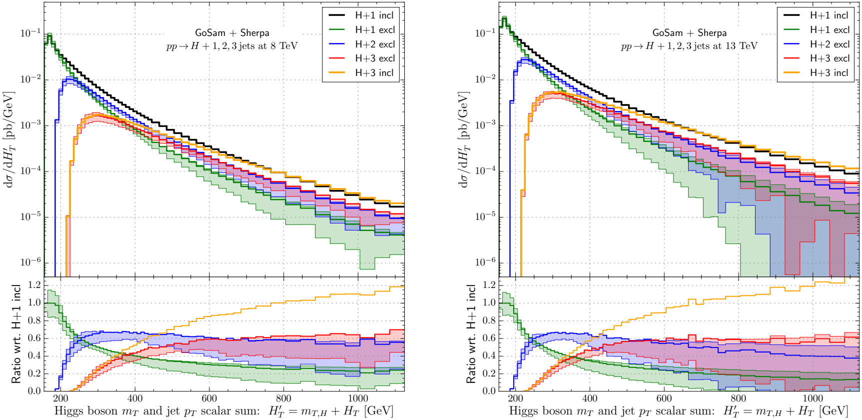


- H_T' – based scale
- Small uncertainties

- m_H – based scale
- Large uncertainties

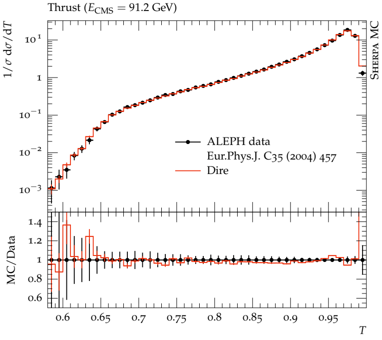
Poor performance example: Jet cuts

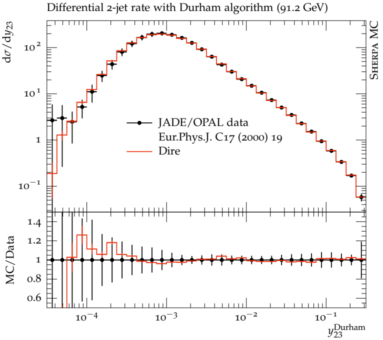
[Greiner et al.] arXiv:1506.01016



- 8 TeV cms energy
- 13 TeV cms energy

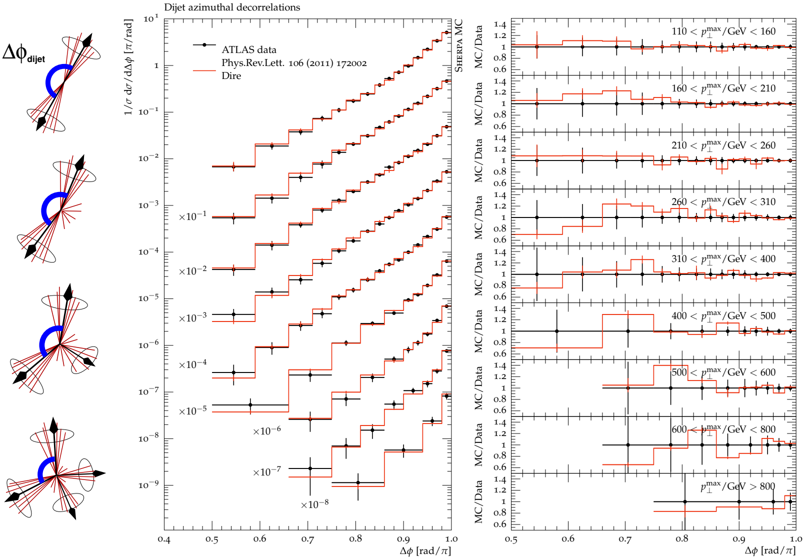
Typical parton shower performance: $e^+e^- \to$jets





- Thrust and Durham $2 \to 3$-jet rate in $e^+e^- \to$hadrons
- Hadronization region to the right (left) in left (right) plot

Typical parton-shower performance: Jets at LHC



Why parton showers work

- Classical point charge on trajectory $y^\mu(s) \to$ conserved current j^μ(x)

$$j^\mu(x) = g \int \mathrm{d}t \, \frac{dy^\mu(t)}{dt} \, \delta^{(4)}(x - y(t)) \, , \qquad g = \sqrt{4\pi\alpha}$$

- Fourier transform to momentum space

$$j^\mu(k) = \int \mathrm{d}^4 x \, e^{ikx} \, j^\mu(x) = g \int \mathrm{d}t \, \frac{dy^\mu(t)}{dt} \, e^{iky(t)}$$

- Assume particle moves with momentum p_a if $t < 0$,
  is 'kicked' at origin $y^\mu(0) = 0$, and moves with p_b if $t > 0$

$$y^\mu(t) = t \, \frac{p^\mu(t)}{p_0(t)} = \begin{cases} t \, p_a^\mu / p_{a,0} & \text{if} \quad t < 0 \\ t \, p_b^\mu / p_{b,0} & \text{if} \quad t > 0 \end{cases}$$

- Introduce a regulator and Fourier transform ...

$$j^\mu(k) = g \int_{-\infty}^{0} \mathrm{d}t \, \frac{p_a^\mu}{p_{a,0}} \, \exp\left\{ i \left(\frac{p_a k}{p_{a,0}} - i\varepsilon \right) t \right\} + g \int_{0}^{+\infty} \mathrm{d}t \, \frac{p_b^\mu}{p_{b,0}} \, \exp\left\{ i \left(\frac{p_b k}{p_{b,0}} + i\varepsilon \right) t \right\}$$

Why parton showers work

- Classical current

$$j^\mu(k) = \; ig\left(\frac{p_b^\mu}{p_b k + i\varepsilon} - \frac{p_a^\mu}{p_a k - i\varepsilon}\right)$$

  - Spin independent
  - Conserved

- Now add the quantum part $\to$ current can create gauge bosons
  Interaction Hamiltonian density

$$\mathcal{H}_{\rm int}(x) = j^\mu(x) A_\mu(x)$$

- Probability of no emission $\to$ vacuum persistence amplitude squared

$$|W_{a\to b}|^2 = |\langle 0|T\left[\exp\left\{i\int \mathrm{d}^4x\, j^\mu(x)A_\mu(x)\right\}\right]|0\rangle|^2$$

- Can be expanded into power series

$$W_{a\to b} = \sum \frac{1}{n!}\, W^{(n)}_{a\to b}\;, \qquad\qquad W^{(n)}_{a\to b} \propto g^n$$

  - Zeroth order: $W^{(0)}_{a\to b} = 1$
  - First order: $\langle 0|A_\mu(x)|0\rangle = 0$

Why parton showers work

- Second order contribution

$$W^{(2)}_{a\to b} = -\int \mathrm{d}^4x \int \mathrm{d}^4y\, j^\mu(x) j^\nu(y) \langle 0|T\left[A_\mu(x)A_\nu(y)\right]|0\rangle$$
$$= -\int \mathrm{d}^4x \int \mathrm{d}^4y\, j^\mu(x) i\Delta_{F,\mu\nu}(x,y) j^\nu(y)$$

  - Emission of field quantum at x, propagation to y & absorption
  - Unobserved, i.e. a *virtual* correction

- Propagation described by time-ordered Green's function

$$i\Delta^{\mu\nu}_F(x,y) = \Theta(y_0-x_0)\langle 0|A^\nu(y)A^\mu(x)|0\rangle + \Theta(x_0-y_0)\langle 0|A^\mu(x)A^\nu(y)|0\rangle$$
$$= \int \frac{\mathrm{d}^3\vec{k}}{(2\pi)^3\,2E_k}\Big[\,\Theta(y_0-x_0)e^{-ik(y-x)} + \Theta(x_0-y_0)e^{ik(y-x)}\Big]\sum_{\lambda=\pm}\varepsilon^\mu_\lambda(k,l)\varepsilon^{\nu\,*}_\lambda(k,l)$$
$$= -i\int \frac{\mathrm{d}^4k}{(2\pi)^4}\frac{e^{-ik(y-x)}}{k^2+i\varepsilon}\sum_{\lambda=\pm}\varepsilon^\mu_\lambda(k)\varepsilon^{\nu\,*}_\lambda(k)$$

Why parton showers work

- Insert into vacuum persistence amplitude

$$W^{(2)}_{a\to b} = -\,i\int \mathrm{d}^4x \int \mathrm{d}^4y \int \frac{\mathrm{d}^4k}{(2\pi)^4}\,\frac{e^{-ik(y-x)}}{k^2+i\varepsilon} \sum_{\lambda=\pm} \big(j(x)\varepsilon_\lambda(k)\big)\big(j(y)\varepsilon_\lambda(k)\big)^*$$

$$= -\,i\int \frac{\mathrm{d}^4k}{(2\pi)^4}\,\frac{1}{k^2+i\varepsilon} \sum_{\lambda=\pm} \big(j(k)\varepsilon_\lambda(k)\big)\big(j(k)\varepsilon_\lambda(k)\big)^*$$

- Use completeness relation for polarization vectors (e.g. axial gauge)

$$\sum_{\lambda=\pm} \varepsilon^\mu_\lambda(k,l)\,\varepsilon^{\nu\,*}_\lambda(k,l) \;=\; -g^{\mu\nu} + \frac{k^\mu l^\nu + k^\nu l^\mu}{kl}$$

- Complete second-order contribution ($p_a^2 = p_b^2 = 0$, dim.reg., $\overline{\mathrm{MS}}$)

$$W^{(2)}_{a\to b} = -\,i\,|g|^2\left(\frac{\mu^2 e^{\gamma_E}}{4\pi}\right)^{\varepsilon} \int \frac{\mathrm{d}^Dk}{(2\pi)^D}\,\frac{1}{k^2+i\varepsilon}\,\frac{2p_ap_b}{(p_ak)(p_bk)}$$

$$\xrightarrow{\text{IR only}} \; -\,\frac{\alpha}{\pi}\left(\frac{1}{\varepsilon^2} - \frac{1}{\varepsilon}\log\frac{2p_ap_b}{\mu^2} + \frac{1}{2}\log^2\frac{2p_ap_b}{\mu^2} - \frac{\pi^2}{12} + \mathcal{O}(\varepsilon)\right)$$

Why parton showers work

- Real-emission contribution

$$\mathrm{d}W^2_{a\to bc}(p_c) = \frac{\mathrm{d}^3\vec{p}_c}{(2\pi)^3\,2E_c}\left|\langle\vec{p}_c|T\left[\exp\left\{i\int\mathrm{d}^4x\,j^\mu(x)A_\mu(x)\right\}\right]|0\rangle\right|^2 .$$

- Can be expanded into power series

$$\mathrm{d}W_{a\to bc}(p_c) = \sum\frac{1}{n!}\mathrm{d}W^{(n)}_{a\to bc}(p_c)\,, \qquad \mathrm{d}W^{(n)}_{a\to bc}(p_c)\propto g^n$$

  - Zeroth order: $\langle\vec{p}_c|0\rangle = 0$

- First-order term ($p_a^2 = p_b^2 = 0$, dim.reg., $\overline{\mathrm{MS}}$)

$$\begin{aligned}
\int\mathrm{d}W^{2\,(1)}_{a\to bc}(p_c) &= \int\frac{\mathrm{d}^3\vec{p}_c}{(2\pi)^3\,2E_c}\left|\,i\int\mathrm{d}^4x\,j^\mu(x)\langle\vec{p}_c|A_\mu(x)|0\rangle\right|^2\\
&= -\int\frac{\mathrm{d}^3\vec{p}_c}{(2\pi)^3\,2E_c}\sum_{\lambda=\pm}\big(j(p_c)\varepsilon_\lambda(p_c)\big)\big(j(p_c)\varepsilon_\lambda(p_c)\big)^*\\
&\to |g|^2\left(\frac{\mu^2e^{\gamma_E}}{4\pi}\right)^\varepsilon\int\frac{\mathrm{d}^D\vec{p}_c}{(2\pi)^D}\,\frac{2p_ap_b}{(p_ap_c)(p_bp_c)}\,\delta(p_c^2)\\
&\approx +\frac{\alpha}{\pi}\left(\frac{1}{\varepsilon^2}-\frac{1}{\varepsilon}\log\frac{2p_ap_b}{\mu^2}+\frac{1}{2}\log^2\frac{2p_ap_b}{\mu^2}-\frac{\pi^2}{12}+\mathcal{O}(\varepsilon)\right)
\end{aligned}$$

Why parton showers work

- So far we have

$$W^{(2)}_{a\to b} = -\frac{\alpha}{\pi}\left(\frac{1}{\varepsilon^2}-\frac{1}{\varepsilon}\log\frac{2p_ap_b}{\mu^2}+\frac{1}{2}\log^2\frac{2p_ap_b}{\mu^2}-\frac{\pi^2}{12}+\mathcal{O}(\varepsilon)\right)$$

$$\int \mathrm{d}W^{2\,(1)}_{a\to bc}(p_c) = +\frac{\alpha}{\pi}\left(\frac{1}{\varepsilon^2}-\frac{1}{\varepsilon}\log\frac{2p_ap_b}{\mu^2}+\frac{1}{2}\log^2\frac{2p_ap_b}{\mu^2}-\frac{\pi^2}{12}+\mathcal{O}(\varepsilon)\right)$$

  - Explicit form of unitarity condition (probability conservation)
  - Poles in ε cancel between virtual and real-emission correction
  - π^2 contributions due to D-dimensional phase space
- Double poles in ε only appear upon integration over loop momentum and full real-emission phase space $\to$ associated with unobserved region $\to$ cancellation between real and virtual (Bloch-Nordsieck / KLN)
- Remaining terms are double logarithms

$$W^{(2)}_{a\to b} \to -\frac{\alpha}{\pi}\left(\frac{1}{2}\log^2\frac{2p_ap_b}{\mu^2}-\frac{\pi^2}{12}+\mathcal{O}(\varepsilon)\right)$$

$$\int \mathrm{d}W^{2\,(1)}_{a\to bc}(p_c) \to +\frac{\alpha}{\pi}\left(\frac{1}{2}\log^2\frac{2p_ap_b}{\mu^2}-\frac{\pi^2}{12}+\mathcal{O}(\varepsilon)\right)$$

  - Terms of this type survive if unitarity is broken by the measurement e.g. vetoed real radiation above a certain scale $\tau\mu^2$
    $\nearrow$ Computation of thrust at NLO in Melissa's lecture

Why parton showers work

- Order 2n contribution to vacuum persistence amplitude

$$W^{(2n)}_{a\to b} = \Big[\prod_{i=1}^{2n} i\int \mathrm{d}^4 x_i j^{\mu_i}(x_i)\Big]\,\langle 0|T\Big[\prod_{i=1}^{2n} A_{\mu_i}(x_i)\Big]|0\rangle$$

- Decompose time-ordered product into Feynman propagators, use symmetry of integrand in currents

$$\frac{W^{(2n)}_{a\to b}}{(2n)!} = \frac{(2n-1)(2n-3)\ldots 3\cdot 1}{(2n)!}\Big[\prod_{i=1}^{2n} i\int \mathrm{d}^4 x_i j^{\mu_i}(x_i)\Big]$$

$$\times\prod_{i=1}^{n}\langle 0|T\left[A_{\mu_{2i}}(x_{2i})A_{\mu_{2i+1}}(x_{2i+1})\right]|0\rangle$$

$$= \frac{1}{2^n n!}\left(-\int \mathrm{d}^4 x\int \mathrm{d}^4 y\, j^\mu(x) i\Delta_{\mu\nu}(x,y) j^\nu(y)\right)^n = \frac{1}{n!}\left(\frac{W^{(2)}_{a\to b}}{2}\right)^n .$$

- Sum all orders in α $\to$ vacuum persistence amplitude squared

$$|W_{a\to b}|^2 = \left|\sum_{n=0}^{\infty}\frac{1}{n!}\left(\frac{W^{(2)}_{a\to b}}{2}\right)^n\right|^2 = \exp\left\{W^{(2)}_{a\to b}\right\} .$$

Why parton showers work

- **Sudakov factor from first principles**

  $$\Delta = |W_{a\to b}|^2 = \exp\left\{W^{(2)}_{a\to b}\right\}$$

  - Resummed virtual corrections at scale μ^2
  - Logarithmic structure same as real corrections

- **For Abelian theories we can also use**

  $$\Delta = \exp\left\{-\int \mathrm{d}W^{2\,(1)}_{a\to bc}\right\}$$

  - Agrees with heuristics based on probability conservation
  - Sufficient for most use cases in non-Abelian theories, but not exact

- Universal, semi-classical integrand (Eikonal)

  $$\frac{2p_ap_b}{(p_ap_c)(p_bp_c)}$$

  - Originates in gauge boson radiation off conserved charge
  - **This is the same for charged particles in full scalar QED / QCD**
    $\rightarrow$ **Kinematical approximations not actually needed**
  - As they use exact phase space factorization,
    parton showers only miss spin effects & correlations

When parton showers don't work ...

... one of the underlying assumptions must be violated

- Relevant phase space is not covered by the evolution
- Multi-parton correlations are resolved by observable
- Scale hierarchies are small (e.g. similar jet-p_T)
- Rapidity differences are large
- Reasons too complicated to explain here ...

If none of these apply and your parton shower still gives nonsense:

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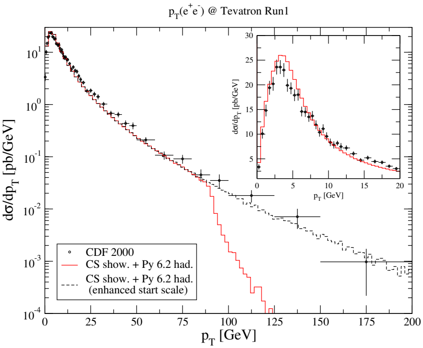
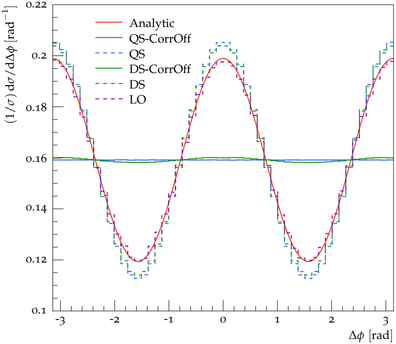
Matching & merging won't solve it.
You should talk to the authors!

Poor performance example: Phase space coverage



- Drell-Yan lepton pair production at Tevatron
- If hard cross section computed at leading order, then parton shower is only source of transverse momentum $\nearrow$ Melissa's lecture
- **Does not generate transverse momenta larger than μ_F**

Poor performance example: Spin correlations



- Azimuthal modulation of QCD radiation due to spin of intermediate gluons

Theory background

Toy model for infrared subtraction at NLO

[Frixione,Webber] hep-ph/0204244

- Assume system of charges radiating "photons" of fractional energy x.
- Predicting observables at NLO amounts to computing expectation value

$$\langle O \rangle = \lim_{\varepsilon \to 0} \int_0^1 \mathrm{d}x\, x^{-2\varepsilon} \left[\left(\frac{\mathrm{d}\sigma}{\mathrm{d}x}\right)_{\mathrm{B}} O_0 + \left(\frac{\mathrm{d}\sigma}{\mathrm{d}x}\right)_{\mathrm{V}} O_0 + \left(\frac{\mathrm{d}\sigma}{\mathrm{d}x}\right)_{\mathrm{R}} O_1(x) \right]$$

- Born, virtual and real-emission contributions given by

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}x}\right)_{\mathrm{B,V,R}} = \quad \mathrm{B}\,\delta(x), \qquad \left(\mathrm{V}_f + \frac{\mathrm{BV}_s}{2\varepsilon}\right)\delta(x), \qquad \frac{\mathrm{R}(x)}{x}$$

KLN cancellation theorem: $\lim_{x\to 0} \mathrm{R}(x) = \mathrm{BV}_s$
Infrared safe observable: $\lim_{x\to 0} O_1(x) = O_0$

Virtual correction $\begin{cases} \mathrm{V}_f & - \quad \text{finite piece} \\ \mathrm{BV}_s/2\varepsilon & - \quad \text{singular piece} \end{cases}$

Implicit: All higher-order terms proportional to coupling α

Toy model for infrared subtraction at NLO

- Add and subtract approximation of real correction in soft limit

$$\langle O\rangle_R = \mathrm{BV}_s\, O(0) \int_0^1 \mathrm{d}x\, \frac{x^{-2\varepsilon}}{x} + \int_0^1 \mathrm{d}x\, \frac{\mathrm{R}(x)\, O(x) - \mathrm{BV}_s\, O(0)}{x^{1+2\varepsilon}}$$

- Second integral non-singular $\rightarrow$ set $\varepsilon = 0$

$$\langle O\rangle_R = -\frac{\mathrm{BV}_s}{2\varepsilon}\, O(0) + \int_0^1 \mathrm{d}x\, \frac{\mathrm{R}(x)\, O(x) - \mathrm{BV}_s\, O(0)}{x}$$

- Combine everything with Born and virtual correction

$$\langle O\rangle = \left(\mathrm{B} + \mathrm{V}_f\right) O(0) + \int_0^1 \frac{\mathrm{d}x}{x} \left[\mathrm{R}(x)\, O(x) - \mathrm{BV}_s\, O(0)\right]$$

  Both terms separately finite as $x \rightarrow 0$

- Rewrite for future reference

$$\langle O\rangle = \left(\mathrm{B} + \mathrm{V} + \mathrm{I}\right) O(0) + \int_0^1 \frac{\mathrm{d}x}{x} \left[\mathrm{R}(x)\, O(x) - \mathrm{S}\, O(0)\right]$$

  $\mathrm{I} = -\mathrm{BV}_s/2\varepsilon \rightarrow$ Integrated subtraction term
  $\mathrm{S} = \mathrm{BV}_s \rightarrow$ Real subtraction term

Actual infrared subtraction at NLO

- QCD subtraction more cumbersome:
  - Soft limit color dependent [Bassetto,Ciafaloni,Marchesini] PR100(1983)201

$$|\mathcal{M}(1,\ldots,j,\ldots,n)|^2 \overset{j\to\text{soft}}{\longrightarrow} -\sum_{i,k\neq i} \frac{8\pi\mu^{2\varepsilon}\alpha_s}{p_i p_j}$$

$$\times\, {}_m\langle 1,\ldots,i,\ldots,k,\ldots,n|\, \frac{\mathbf{T}_i\mathbf{T}_k\; p_i p_k}{(p_i+p_k)p_j}\, |1,\ldots,i,\ldots,k,\ldots,n\rangle_m$$

$\mathbf{T}_i$ - color insertion operator for parton i

$|1,\ldots,i,\ldots,k,\ldots,n\rangle_m$ - m-parton Born amplitude

  - Collinear limit spin dependent [Altarelli,Parisi] NPB126(1977)298

$$|\mathcal{M}(1,\ldots,i,\ldots,j,\ldots,n)|^2 \overset{i,j\to\text{coll}}{\longrightarrow} \frac{8\pi\mu^{2\varepsilon}\alpha_s}{2p_i p_j}$$

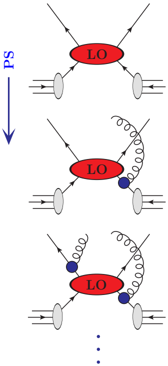
$$\times\, {}_m\langle 1,\ldots,ij,\ldots,n|\hat{P}_{(ij)i}(z,k_T,\varepsilon)\,|1,\ldots,ij,\ldots,n\rangle_m$$

$\hat{P}_{(ij)i}(z,k_T,\varepsilon)$ - Spin-dependent DGLAP kernel

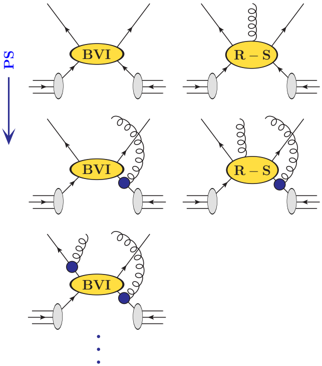
- Basic features surviving from toy model are phase-space mapping and subtraction terms as products of Born times splitting operator
- Commonly used techniques: Dipole method & FKS method

[Catani,Seymour] NPB485(1997)291, [Catani,Dittmaier,Seymour,Trocsanyi] NPB627(2002)189
[Frixione,Kunszt,Signer] NPB467(1996)399

NLO matching



NLO matching



Matching schemes

Two major techniques to match NLO calculations and parton showers

Additive (MC@NLO-like)

[Frixione,Webber] hep-ph/0204244

- Use parton-shower splitting kernel as an NLO subtraction term
- Multiply LO event weight by Born-local K-factor including integrated subtraction term and virtual corrections
- Add hard remainder function consisting of subtracted real-emission correction

Multiplicative (POWHEG-like)

[Nason] hep-ph/0409146

- Use matrix-element corrections to replace parton-shower splitting kernel by full real-emission matrix element in first shower branching
- Multiply LO event weight by Born-local NLO K-factor (integrated over real corrections that can be mapped to Born according to PS kinematics)

Toy model for modified subtraction

[Frixione,Webber] hep-ph/0204244

- Revisit toy model for NLO

$$\langle O \rangle = \Big(\mathrm{B} + \mathrm{V} + \mathrm{I}\Big)\, O(0) + \int_0^1 \frac{\mathrm{d}x}{x} \Big[\mathrm{R}(x)\, O(x) - \mathrm{S}\, O(0)\Big]$$

- In parton showers, any number of “photons” can be emitted
- Emission probability controlled by Sudakov form factor

$$\Delta(x_1, x_2) = \exp\left\{ -\int_{x_1}^{x_2} \frac{\mathrm{d}x}{x}\, \mathrm{K}(x) \right\}$$

Evolution kernel behaves as $\lim_{x\to 0} \mathrm{K}(x) = \lim_{x\to 0} \mathrm{R}(x)/\mathrm{B} = \mathrm{V}_s$

- Define generating functional

$$\mathcal{F}^{(n)}_{\mathrm{MC}}(x, O) = \Delta(x_0, x)\, O_n(x) + \int_{x_0}^{x} \frac{\mathrm{d}\bar{x}}{\bar{x}} \frac{\mathrm{d}\Delta(\bar{x}, x)}{\mathrm{d}\ln\bar{x}}\, \mathcal{F}^{(n+1)}_{\mathrm{MC}}(\bar{x}, O)$$

- $\mathcal{F}^{(n)}_{\mathrm{MC}}(x, O)$ now replaces observable $O \to$ Naively:

$O(0) \Leftrightarrow$ start MC with 0 emissions $\to \mathcal{F}^{(0)}_{\mathrm{MC}}(1, O)$

$O(x) \Leftrightarrow$ start MC with 1 emission $\to \mathcal{F}^{(1)}_{\mathrm{MC}}(x, O)$

Toy model for modified subtraction

- Combined generating functional would be

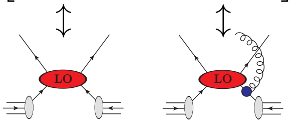
$$\left[\Big(\mathrm{B}+\mathrm{V}+\mathrm{I}\Big)-\int_0^1\frac{\mathrm{d}x}{x}\mathrm{S}\right]\mathcal{F}_{\mathrm{MC}}^{(0)}(1,O)+\int_0^1\frac{\mathrm{d}x}{x}\,\mathrm{R}(x)\,\mathcal{F}_{\mathrm{MC}}^{(1)}(x,O)$$

- This is wrong because

$$\mathcal{F}_{\mathrm{MC}}^{(0)}(O)=\Delta(x_c,1)\,O(0)+\int_{x_c}^1\frac{\mathrm{d}x}{x}\mathrm{K}(x)\Delta(x,1)\,O(x)+\ldots$$

- So $\mathrm{B}\,\mathcal{F}_{\mathrm{MC}}^{(0)}$ generates an $\mathcal{O}(\alpha)$ term that spoils NLO accuracy

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}x}\right)_{\mathrm{MC}}O(x)=\mathrm{B}\left[-\frac{\mathrm{K}(x)}{x}\,O(0)+\frac{\mathrm{K}(x)}{x}\,O(x)\right]$$



Toy MC@NLO

[Frixione,Webber] hep-ph/0204244

- The proper matching is obtained by subtracting this $\mathcal{O}(\alpha)$ contribution

$$\langle O \rangle = \Bigg[\underbrace{\Big(\mathrm{B} + \mathrm{V} + \mathrm{I}\Big) + \int_0^1 \frac{\mathrm{d}x}{x} \Big(\mathrm{BK}(x) - \mathrm{S}\Big)}_{\text{NLO-weighted Born cross section}} \Bigg] \mathcal{F}_{\mathrm{MC}}^{(0)}(1, O)$$

$$+ \int_0^1 \frac{\mathrm{d}x}{x} \underbrace{\Big[\mathrm{R}(x) - \mathrm{BK}(x)\Big]}_{\text{hard remainder}} \mathcal{F}_{\mathrm{MC}}^{(1)}(x, O)$$

- Like at fixed order, both terms are separately finite
- We call events from the first term $\mathbb{S}$-events ($\mathbb{S}$tandard) and events from the second term $\mathbb{H}$-events ($\mathbb{H}$ard)
- For further reference, define $\mathrm{D}^{(\mathrm{K})}(x) := \mathrm{BK}(x)$ as well as

$$\bar{\mathrm{B}}^{(\mathrm{K})} = \Big(\mathrm{B} + \mathrm{V} + \mathrm{I}\Big) + \int_0^1 \frac{\mathrm{d}x}{x} \Big(\mathrm{D}^{(\mathrm{K})}(x) - \mathrm{S}\Big) , \qquad \mathrm{H}^{(\mathrm{K})}(x) = \mathrm{R}(x) - \mathrm{D}^{(\mathrm{K})}(x)$$

$\rightarrow$ compact notation

$$\langle O \rangle = \bar{\mathrm{B}}^{(\mathrm{K})} \mathcal{F}_{\mathrm{MC}}^{(0)}(O) + \int_0^1 \frac{\mathrm{d}x}{x} \mathrm{H}^{(\mathrm{K})}(x) \, \mathcal{F}_{\mathrm{MC}}^{(1)}(x, O)$$

Modified subtraction in QCD

[Frixione,Webber] hep-ph/0204244

- Leading-order calculation for observable O

$$\langle O\rangle = \int \mathrm{d}\Phi_B\,\mathrm{B}(\Phi_B)\,O(\Phi_B)$$

- NLO calculation for same observable

$$\langle O\rangle = \int \mathrm{d}\Phi_B\left\{\mathrm{B}(\Phi_B)+\tilde{V}(\Phi_B)\right\}O(\Phi_B)+\int \mathrm{d}\Phi_R\,\mathrm{R}(\Phi_R)\,O(\Phi_R)$$

- Parton-shower result until first emission

$$\langle O\rangle = \int \mathrm{d}\Phi_B\,\mathrm{B}(\Phi_B)\left[\Delta^{(\mathrm{K})}(t_c)\,O(\Phi_B)+\int_{t_c}\mathrm{d}\Phi_1\,\mathrm{K}(\Phi_1)\,\Delta^{(\mathrm{K})}(t(\Phi_1))\,O(\Phi_R)\right]$$

$$\overset{\mathcal{O}(\alpha_s)}{\rightarrow}\int \mathrm{d}\Phi_B\,\mathrm{B}(\Phi_B)\left\{1-\int_{t_c}\mathrm{d}\Phi_1\mathrm{K}(\Phi_1)\right\}O(\Phi_B)+\int_{t_c}\mathrm{d}\Phi_B\mathrm{d}\Phi_1\,\mathrm{B}(\Phi_B)\,\mathrm{K}(\Phi_1)\,O(\Phi_R)$$

Phase space: $\mathrm{d}\Phi_1 = \mathrm{d}t\,\mathrm{d}z\,\mathrm{d}\phi$

Splitting functions: $\mathrm{K}(t,z)\rightarrow \alpha_s/(2\pi t)\sum \mathrm{P}(z)\,\Theta(\mu_Q^2-t)$

Sudakov factors: $\Delta^{(\mathrm{K})}(t) = \exp\left\{-\int_t \mathrm{d}\Phi_1\mathrm{K}(\Phi_1)\right\}$

Modified subtraction in QCD

- Subtract $\mathcal{O}(\alpha_s)$ PS terms from NLO result

$$\int \mathrm{d}\Phi_B \left\{ \mathrm{B}(\Phi_B) + \tilde{\mathrm{V}}(\Phi_B) + \mathrm{B}(\Phi_B) \int \mathrm{d}\Phi_1 \mathrm{K}(\Phi_1) \right\} \ldots$$

$$+ \int \mathrm{d}\Phi_R \left\{ \mathrm{R}(\Phi_R) - \mathrm{B}(\Phi_B)\,\mathrm{K}(\Phi_1) \right\} \ldots$$

- In DLL approximation both terms finite $\rightarrow$
  MC events in two categories, $\mathbb{S}$tandard and $\mathbb{H}$ard

$$\mathbb{S} \;\rightarrow\; \bar{\mathrm{B}}^{(\mathrm{K})}(\Phi_B) = \mathrm{B}(\Phi_B) + \tilde{\mathrm{V}}(\Phi_B) + \mathrm{B}(\Phi_B) \int \mathrm{d}\Phi_1 \mathrm{K}(\Phi_1)$$

$$\mathbb{H} \;\rightarrow\; \mathrm{H}^{(\mathrm{K})} = \mathrm{R}(\Phi_R) - \mathrm{B}(\Phi_B)\mathrm{K}(\Phi_1)$$

- Color & spin correlations $\rightarrow$ NLO subtraction needed
  $1/N_c$ corrections can be faded out in soft region by smoothing function

$$\bar{\mathrm{B}}^{(\mathrm{K})}(\Phi_B) = \mathrm{B}(\Phi_B) + \tilde{\mathrm{V}}(\Phi_B) + \mathrm{I}(\Phi_B) + \int \mathrm{d}\Phi_1 \Big[\mathrm{B}(\Phi_B)\,\mathrm{K}(\Phi_1) - \mathrm{S}(\Phi_R) \Big] f(\Phi_1)$$

$$\mathrm{H}^{(\mathrm{K})}(\Phi_R) = \Big[\mathrm{R}(\Phi_R) - \mathrm{B}(\Phi_B)\,\mathrm{K}(\Phi_1) \Big] f(\Phi_1)$$

Dealing with color and spin

Method 1 [Frixione,Webber] hep-ph/0204244

- $f(\Phi_1) \to 0$ in soft-gluon limit
- Full NLO in hard / collinear region
- Subleading color terms not ϕ_1-dependent in soft domain

Method 2 [Krauss,Schönherr,Siegert,SH] arXiv:1111.1220

- Replace $\mathrm{B}(\Phi_B)\mathrm{K}(\Phi_1) \to \mathrm{S}(\Phi_R)$, includes color & spin correlations
- Can lead to non-probabilistic $\Delta^{(\mathrm{S})}(t)$
  $\to$ requires modification of veto algorithm

MC@NLO

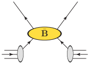
[Frixione,Webber] hep-ph/0204244

- Add parton shower, described by generating functional $\mathcal{F}_{\mathrm{MC}}$

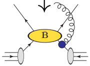
$$\langle O \rangle = \int \mathrm{d}\Phi_B \, \bar{\mathrm{B}}^{(\mathrm{K})}(\Phi_B) \, \mathcal{F}^{(0)}_{\mathrm{MC}}(\mu_Q^2, O) + \int \mathrm{d}\Phi_R \, \mathrm{H}^{(\mathrm{K})}(\Phi_R) \, \mathcal{F}^{(1)}_{\mathrm{MC}}(t(\Phi_R), O)$$

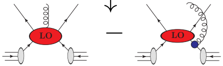
  Probability conservation: $\mathcal{F}_{\mathrm{MC}}(t, 1) = 1 \rightarrow$ cross section correct at NLO
- Expansion of matched result until first emission

$$\langle O \rangle = \int \mathrm{d}\Phi_B \bar{\mathrm{B}}^{(\mathrm{K})}(\Phi_B) \Big[\Delta^{(\mathrm{K})}(t_c) \, O(\Phi_B)$$



$$+ \int_{t_c} \mathrm{d}\Phi_1 \mathrm{K}(\Phi_1) \Delta^{(\mathrm{K})}(t(\Phi_1)) \, O(\Phi_r) \Big] + \int \mathrm{d}\Phi_R \, \mathrm{H}^{(\mathrm{K})}(\Phi_{n+1}) \, O(\Phi_R)$$



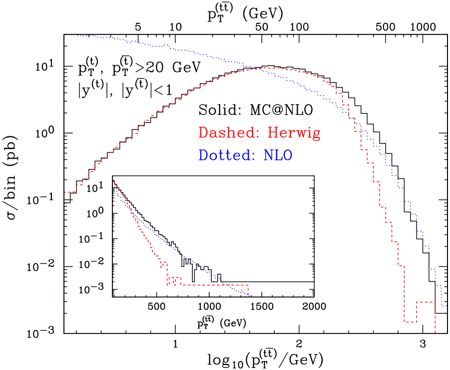


- Parametrically $\mathcal{O}(\alpha_s)$ correct
- Preserves logarithmic accuracy of PS

MC@NLO – Features

[Nason,Webber] arXiv:1202.1251

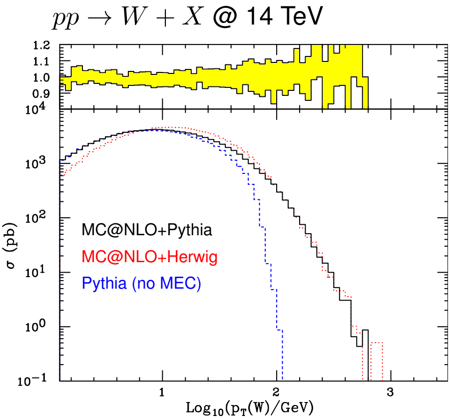
$pp \to t\bar{t} + X$ @ 14 TeV



- MC@NLO interpolates smoothly between real-emission ME and PS

MC@NLO – Features

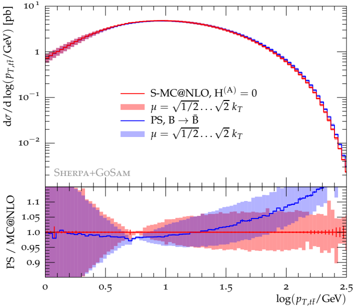
[Torrielli,Frixione] arXiv:1002.4293

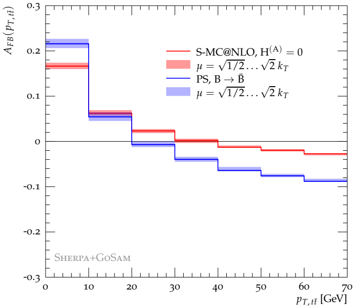


- MC@NLO with different PS agree at high $p_T \leftrightarrow$ NLO
- Differences at low p_T due to differences in PS

MC@NLO – Features

[Huang,Luisoni,Schönherr,Winter,SH] arXiv:1306.2703





- Leading color appropriate for sufficiently inclusive observables
- Full vs leading color has larger impact on A_{FB} $\to$ explained by kinematics effects using arguments of [Skands,Webber,Winter] arXiv:1205.1466

POWHEG

[Nason] hep-ph/0409146

- Aim of the method: Eliminate negative weights from MC@NLO
- Replace $\mathrm{BK} \to \mathrm{R} \Rightarrow$ no $\mathbb{H}$-events $\quad \Rightarrow \quad \bar{\mathrm{B}}^{(\mathrm{R})}$ positive in physical region
- Expectation value of observable is

$$\langle O \rangle = \int \mathrm{d}\Phi_B \bar{\mathrm{B}}^{(\mathrm{R})}(\Phi_B) \Bigg[\Delta^{(\mathrm{R})}(t_c, s_{\mathrm{had}})\, O(\Phi_B) + \int_{t_c}^{s_{\mathrm{had}}} \mathrm{d}\Phi_1 \frac{\mathrm{R}(\Phi_R)}{\mathrm{B}(\Phi_B)} \Delta^{(\mathrm{R})}(t(\Phi_1), s_{\mathrm{had}})\, O(\Phi_R) \Bigg]$$

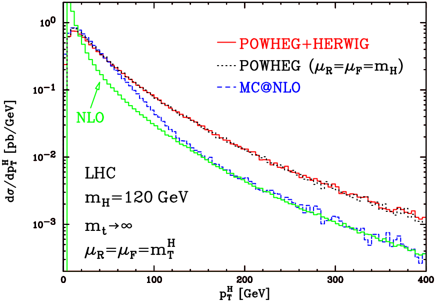
- μ_Q^2 has changed to hadronic centre-of-mass energy squared, s_{had}, as full phase space for real-emission correction, R, must be covered
- Absence of $\mathbb{H}$-events leads to enhancement of high-p_T region by

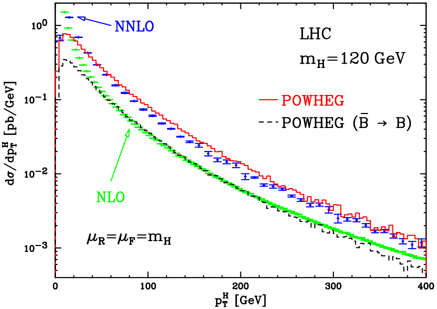
  $$K = \frac{\bar{\mathrm{B}}}{\mathrm{B}} = 1 + \mathcal{O}(\alpha_s)$$

  Formally beyond NLO, but sizeable corrections in practice

POWHEG – Features

[Alioli,Nason,Oleari,Re] arXiv:0812.0578





- Large enhancement at high $p_{T,h}$
- Can be traced back to large NLO correction
- Fortunately, NNLO correction is also large $\to\sim$ agreement

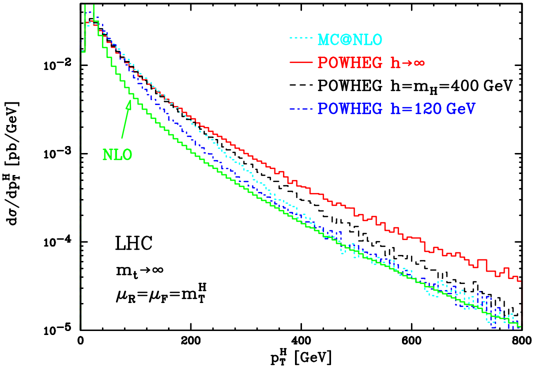
Improved POWHEG

- To avoid problems in high-p_T region, split real-emission ME into singular and finite parts as $\mathrm{R} = \mathrm{R}^s + \mathrm{R}^f$
- Treat singular piece in $\mathbb{S}$-events and finite piece in $\mathbb{H}$-events
  Similar to MC@NLO with redefined PS evolution kernels
- Differential event rate up to first emission

$$\langle O \rangle = \int \mathrm{d}\Phi_B \bar{\mathrm{B}}^{(\mathrm{R}^s)}(\Phi_B) \Bigg[\Delta^{(\mathrm{R}^s)}(t_c, s_{\mathrm{had}})\, O(\Phi_B) + \int_{t_c}^{s_{\mathrm{had}}} \mathrm{d}\Phi_1 \frac{\mathrm{R}^s(\Phi_R)}{\mathrm{B}(\Phi_B)} \Delta^{(\mathrm{R}^s)}(t(\Phi_1), s_{\mathrm{had}})\, O(\Phi_R) \Bigg] + \int \mathrm{d}\Phi_R\, \mathrm{R}^f_n(\Phi_R)$$

POWHEG – Features

[Alioli,Nason,Oleari,Re] arXiv:0812.0578



- Singular real-emission part here defined as

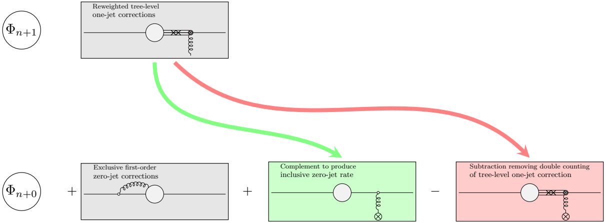
$$\mathrm{R}^s = \mathrm{R}\,\frac{h^2}{p_T^2 + h^2}$$

- Can “tune” NNLO contribution by varying free parameter h

Unitarity-based techniques

[Lönnblad,Prestel] arXiv:1211.4827, [Plätzer] arXiv:1211.5467

U(N)LOPS

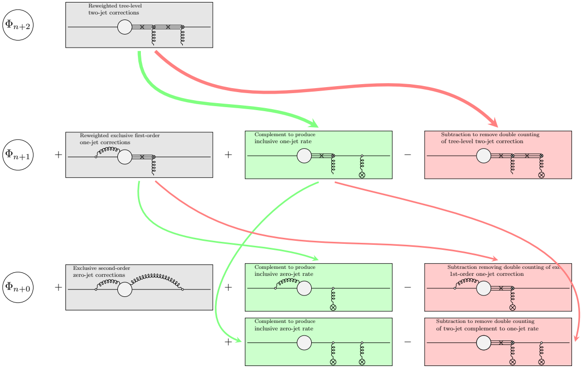


- Compute vetoed cross section & complete with real-emission
- Add Sudakov vetoed real-emission cross section & projection
- Can be implemented based on only two inputs (gray boxes)

Unitarity-based techniques

[Lönnblad,Prestel] arXiv:1211.4827, [Li,Prestel,SH] arXiv:1405.3607

UN^{2}LOPS

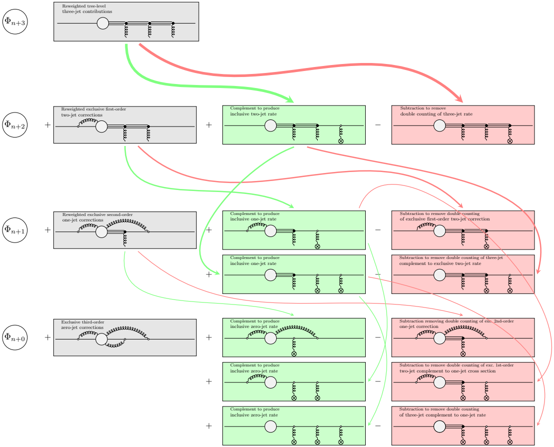


- Same idea as in ULOPS, but now also adding 2-loop contribution

Unitarity-based techniques

[Prestel] arXiv:2106.03206, [Bertone,Prestel] arXiv:2202.01082

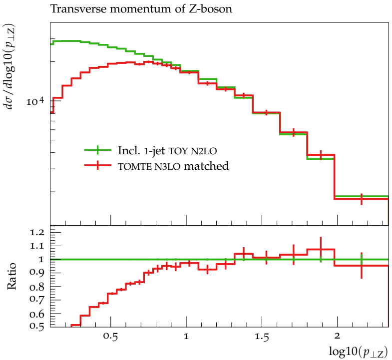
TOMTE

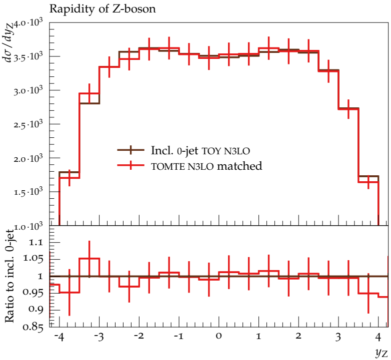


- Same idea as in UN^{2}LOPS, but now also adding 3-loop contribution
- Must pay careful attention to projections (relevant for all UN^{X}LOPS)

TOMTE – Features

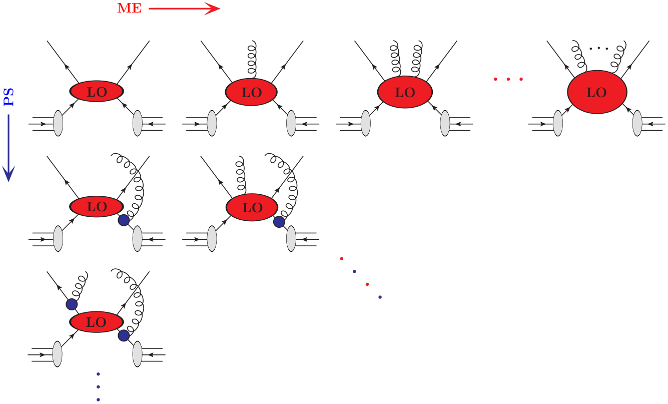
[Bertone,Prestel] arXiv:2202.01082





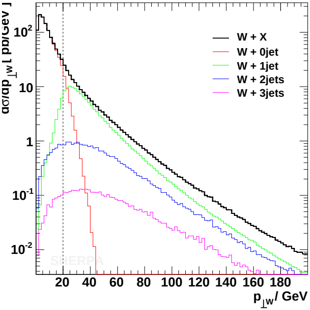
- Drell-Yan lepton pair production at LHC
- Stand-in fixed-order calculation for closure tests

Multi-jet merging



Basic idea of merging

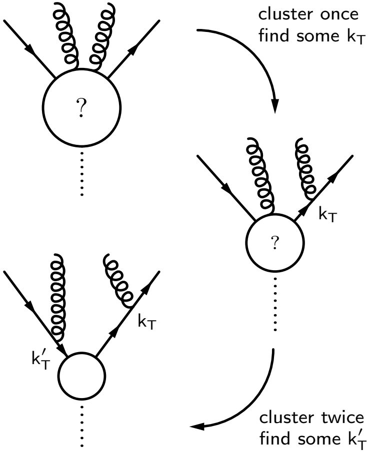
- Separate phase space into "hard" and "soft" region
- Parton shower populates soft domain
- N^{x}LO real corrections replace PS emission term in hard domain
- Need criterion to define "hard" & "soft" $\rightarrow$ jet measure Q and corresponding cut, Q_{cut}



Parton shower histories

[André,Sjöstrand] hep-ph/9708390

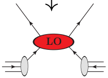
- Start with some "core" process
  for example $e^+e^- \to q\bar{q}$
- This process is considered inclusive
  It sets the resummation scale μ_Q^2
- Higher-multiplicity ME can be
  reduced to core by clustering
  - Identify most likely splitting
    according to PS emission probability
  - Combine partons into mother
    according to PS kinematics
  - Continue until core process reached



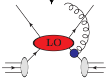
Basic idea of merging

- MC@LO split into $Q < Q_{\rm cut}$ (PS) and $Q > Q_{\rm cut}$ (ME) region
  PS expression replaced by real-emission matrix-element in ME region

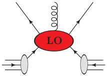
$$\langle O \rangle = \int \mathrm{d}\Phi_B \mathrm{B}(\Phi_B) \bigg[\Delta^{(\mathrm{K})}(t_c, \mu_Q^2)\, O(\Phi_B)$$



$$+ \int_{t_c}^{\mu_Q^2} \mathrm{d}\Phi_1 \mathrm{K}(\Phi_1)\, \Delta^{(\mathrm{K})}(t(\Phi_1), \mu_Q^2)\, \Theta(Q_{\rm cut} - Q)\, O(\Phi_R) \bigg]$$



$$+ \int \mathrm{d}\Phi_R \mathrm{R}(\Phi_R)\, \Delta^{(\mathrm{K})}(t(\Phi_R), \mu_Q^2)\, \Theta(Q - Q_{\rm cut})\, O(\Phi_R)$$



- Jet veto in PS / Jet cut on ME
- To match $\mathrm{K}(\phi_1)$, weight $\mathrm{R}(\phi_1)$ by $\alpha_s(k_T^2)/\alpha_s(\mu_R^2)$

Truncated vetoed parton showers

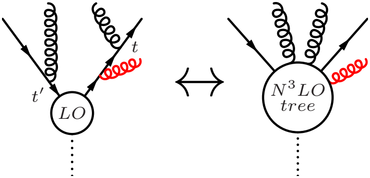
[Lönnblad] hep-ph/0112284

- In hard region $\Delta(t(\Phi_R),\mu_Q^2)$ is additional weight
- Most efficiently computed using pseudo-showers

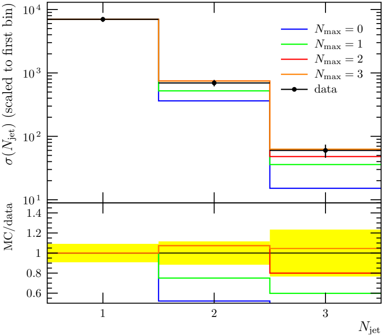
  Recall PS no-emission probability: Constrained: $\Pi(x,t_2,\mu_Q^2)/\Pi(x,t_1,\mu_Q^2)$

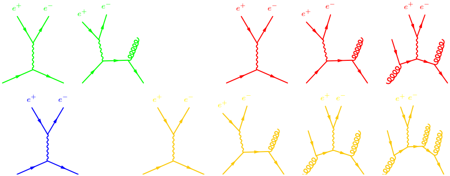
  Unconstrained: $\Delta(t_2,\mu_Q^2)/\Delta(t_1,\mu_Q^2)$

- Start PS from core process
- Evolve until predefined branching
  $\leftrightarrow$ truncated parton shower
- Emissions that would produce additional hard jets
  lead to event rejection (veto)



Effects of merging - Z+jets at the Tevatron

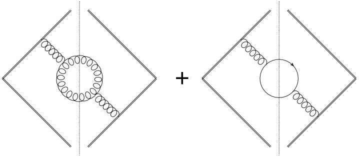




- MC predictions for exclusive n-jet rates match data well as long as corresponding final states are described by matrix elements

Universal higher-order corrections

- Approximate soft-gluon emission times collinear decay in $q(i)\bar{q}(j)g(1)g(2)$ using semi-classical limit and gluon splitting function



$$= \sum_{b=q,g} j_{ij,\mu}(p_{12}) j_{ij,\nu}(p_{12}) \frac{P^{\mu\nu}_{gb}(z_1)}{s_{12}}$$

$$P^{\mu\nu}_{gq}(z) = T_R \left(-g^{\mu\nu} + 4\,z(1-z)\,\frac{k_\perp^\mu k_\perp^\nu}{k_\perp^2} \right)$$

$$P^{\mu\nu}_{gg}(z) = C_A \left(-g^{\mu\nu} \left(\frac{z}{1-z} + \frac{1-z}{z} \right) - 2\,(1-\varepsilon) z(1-z)\,\frac{k_\perp^\mu k_\perp^\nu}{k_\perp^2} \right)$$

- Combine with phase space for one parton emission in collinear limit $D = 4 - 2\varepsilon$, $y = s_{12}/Q^2$, see for example [Catani,Seymour] hep-ph/9605323

$$\mathrm{d}\Phi_{+1} = \frac{Q^{2-2\varepsilon}}{16\pi^2} \frac{(4\pi)^\varepsilon}{\Gamma(1-\varepsilon)}\,\mathrm{d}y\,\mathrm{d}z\,\left[y\,z(1-z)\right]^{-\varepsilon}$$

- Perform Laurent series expansion

$$\frac{1}{y^{1+\varepsilon}} = -\frac{\delta(y)}{\varepsilon} + \sum_{n=0}^{\infty} \frac{\varepsilon^n}{n!} \left(\frac{\ln^n y}{y} \right)_+$$

Universal higher-order corrections

- $\mathcal{O}(\varepsilon^0)$ differential remainder terms have contributions proportional to

$$g \to q\bar{q}: \quad T_R \Big[2z(1-z) + \big(1-2z(1-z)\big)\ln(z(1-z))\Big]$$

$$g \to gg: \quad 2C_A \left[\frac{\ln z}{1-z} + \frac{\ln(1-z)}{z} + \big(-2+z(1-z)\big)\ln(z(1-z))\right]$$

- Integration over z, addition of some semi-classical terms & one-loop soft current gives two-loop cusp anomalous dimension

$$K = \left(\frac{67}{18} - \frac{\pi^2}{6}\right) C_A - \frac{10}{9} T_R n_f$$

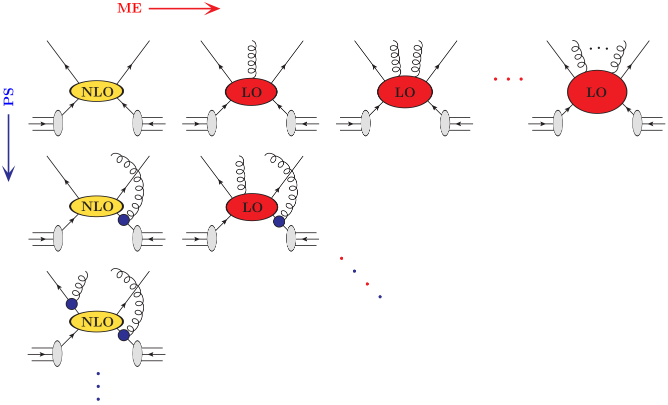
  - Local K-factor for soft-gluon emission
  - Scheme dependent: originates in dim. reg. and $\overline{\mathrm{MS}}$
  - **Can be absorbed in effective coupling** [Catani,Marchesini,Webber] NPB349(1991)635

- Similarly, we find $\mathcal{O}(\varepsilon^0)$ contributions proportional to

$$\frac{\alpha_s}{2\pi}\beta_0 \log \frac{(p_i p_{12})(p_{12} p_j)}{(p_i p_j)\mu^2}$$

  - Can be eliminated by setting scale to transverse mass of soft pair
  - **Leading NLO correction** [Amati, et al.] NPB173(1980)429

Combining Matching and Merging

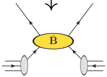


Combined matching and merging with POWHEG

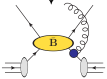
[Hamilton,Nason] arXiv:1004.1764
[Krauss,Schönherr,Siegert,SH] arXiv:1009.1127

- Increase accuracy below Q_{cut} to full NLO

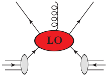
$$\langle O\rangle = \int \mathrm{d}\Phi_B\,\bar{\mathrm{B}}^{(\mathrm{R})}(\Phi_B)\Big[\,\Delta^{(\mathrm{R})}(t_c, s_{\mathrm{had}})\,O(\Phi_B)$$



$$+\int_{t_c}^{s_{\mathrm{had}}} \mathrm{d}\Phi_1\,\frac{\mathrm{R}(\Phi_R)}{\mathrm{B}(\Phi_B)}\,\Delta^{(\mathrm{R})}(t(\Phi_1), s_{\mathrm{had}})\,\Theta(Q_{\mathrm{cut}}-Q)\,O(\Phi_R)\Big]$$



$$+\int \mathrm{d}\Phi_R\,\mathrm{k}^{(\mathrm{R})}(\Phi_R)\,\mathrm{R}(\Phi_R)\,\Delta^{(\mathrm{K})}(t(\Phi_R), \mu_Q^2)\,\Theta(Q-Q_{\mathrm{cut}})\,O(\Phi_R)$$

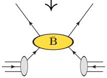


- Local K-factor for smooth merging

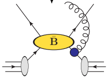
Combined matching and merging with MC@NLO

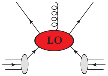
- Increase accuracy below Q_{cut} to full NLO

$$\langle O\rangle = \int \mathrm{d}\Phi_B \bar{\mathrm{B}}^{(\mathrm{K})}(\Phi_B)\Big[\Delta^{(\mathrm{K})}(t_c,\mu_Q^2)\,O(\Phi_B)$$



$$+\int_{t_c}^{\mu_Q^2}\mathrm{d}\Phi_1 \mathrm{K}(\Phi_1)\Delta^{(\mathrm{K})}(t,\mu_Q^2)\Theta(Q_{\mathrm{cut}}-Q)\,O(\Phi_R)\Big] + \int \mathrm{d}\Phi_R\,\mathrm{H}^{(\mathrm{K})}(\Phi_R)\Theta(Q_{\mathrm{cut}}-Q)O(\Phi_R)$$





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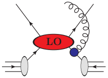
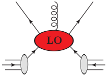


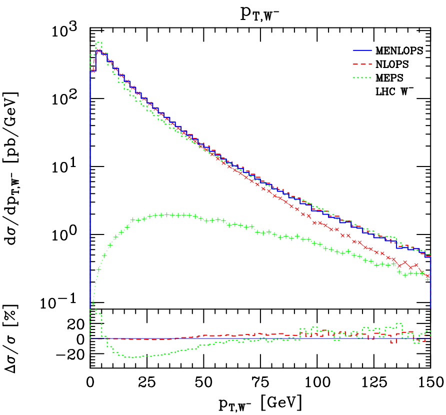
$$+\int \mathrm{d}\Phi_R\,\mathrm{k}^{(\mathrm{K})}(\Phi_R)\mathrm{R}(\Phi_R)\,\Delta^{(\mathrm{K})}(t,\mu_Q^2;>Q_{\mathrm{cut}})\,\Theta(Q-Q_{\mathrm{cut}})\,O(\Phi_R)$$



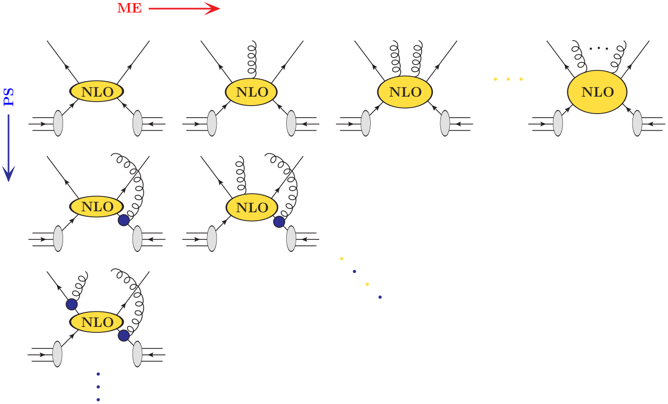
- Local K-factor for smooth merging

Combining matching and merging

[Hamilton,Nason] arXiv:1004.1764



Merging of multiple matched calculations



Merging of multiple matched calculations

- ME+PS merging for 0+1-jet in MC@NLO notation

$$\langle O \rangle = \int \mathrm{d}\Phi_B \mathrm{B}(\Phi_B) \Big[\Delta^{(\mathrm{K})}(t_c)\, O(\Phi_B) + \int_{t_c} \mathrm{d}\Phi_1 \mathrm{K}(\Phi_1)\, \Delta^{(\mathrm{K})}(t)\, \Theta(Q_{\mathrm{cut}} - Q)\, O(\Phi_R) \Big]$$
$$+ \int \mathrm{d}\Phi_R\, \mathrm{R}(\Phi_R)\, \Delta^{(\mathrm{K})}(t(\Phi_R), \mu_Q^2)\, \Theta(Q - Q_{\mathrm{cut}})\, O(\Phi_R)$$

- Reorder by parton multiplicity k, change notation $\mathrm{R}_k \to \mathrm{B}_{k+1}$
- Analyze exclusive contribution from k hard partons only ($t_0 = \mu_Q^2$)

$$\langle O \rangle_k^{\mathrm{excl}} = \int \mathrm{d}\Phi_k\, \mathrm{B}_k \prod_{i=0}^{k-1} \Delta_i^{(\mathrm{K})}(t_{i+1}, t_i)\, \Theta(Q_k - Q_{\mathrm{cut}})$$
$$\times \Big[\Delta_k^{(\mathrm{K})}(t_c, t_k)\, O_k + \int_{t_c}^{t_k} \mathrm{d}\Phi_1\, \mathrm{K}_k\, \Delta_k^{(\mathrm{K})}(t_{k+1}, t_k)\, \Theta(Q_{\mathrm{cut}} - Q_{k+1})\, O_{k+1} \Big]$$

Merging of multiple matched calculations

[Lavesson,Lönnblad,Prestel] arXiv:0811.2912 arXiv:1211.7278
[Gehrmann,Krauss,Schönherr,Siegert,SH] arXiv:1207.5031 arXiv:1207.5030
[Frederix,Frixione] arXiv:1209.6215

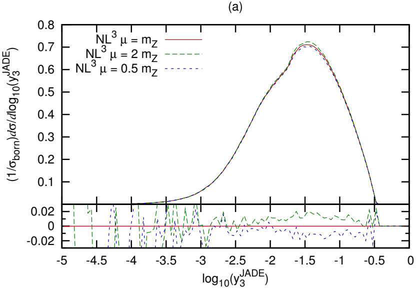
- Analyze exclusive contribution from k hard partons

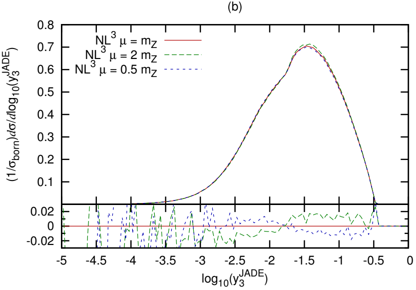
$$
\begin{aligned}
\langle O\rangle_k^{\rm excl} = & \int \mathrm{d}\Phi_k\, \bar{\mathrm{B}}_k^{(\mathrm{K})} \prod_{i=0}^{k-1} \Delta_i^{(\mathrm{K})}(t_{i+1},t_i)\,\Theta(Q_k-Q_{\rm cut}) \\
& \times \left(1+\frac{\mathrm{B}}{\bar{\mathrm{B}}_k^{(\mathrm{K})}}\sum_{i=0}^{k-1}\int_{t_{i+1}}^{t_i}\mathrm{d}\Phi_1 \mathrm{K}_i\,\Theta(Q_i-Q_{\rm cut})+\dots\right) \\
& \times \left[\Delta_k^{(\mathrm{K})}(t_c,t_k)\,O_k+\int_{t_c}^{t_k}\mathrm{d}\Phi_1\;\mathrm{K}_k\,\Delta_k^{(\mathrm{K})}(t_{k+1},t_k)\,\Theta(Q_{\rm cut}-Q_{k+1})\,O_{k+1}\right] \\
& + \int \mathrm{d}\Phi_{k+1}\,\mathrm{H}_k^{(\mathrm{K})}\,\Delta_k^{(\mathrm{K})}(t_k,\mu_Q^2)\,\Theta(Q_k-Q_{\rm cut})\,\Theta(Q_{\rm cut}-Q_{k+1})\,O_{k+1}
\end{aligned}
$$

- Born matrix element $\to$ NLO-weighted Born
- Add hard remainder function
- Subtract $\mathcal{O}(\alpha_s)$ terms from truncated vetoed PS

$e^+e^- \to$ hadrons at LEP

[Lavesson,Lönnblad] arXiv:0811.2912

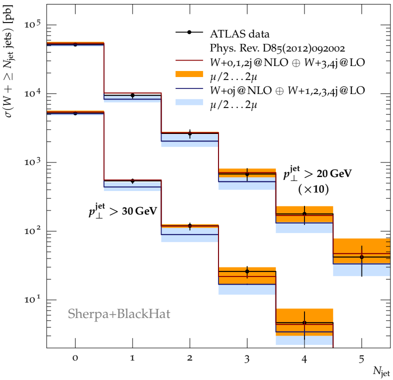


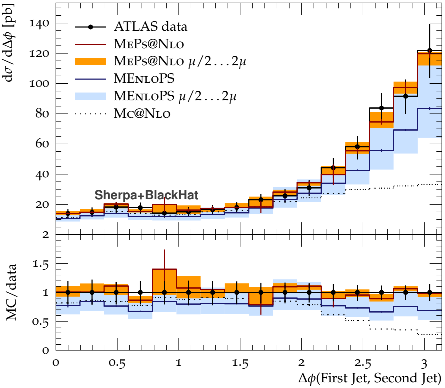


- Scale variations around 2%
- Agreement between 1- and 2-loop but no further reduction of uncertainty

W+jets production at the LHC

[ATLAS] arXiv:1201.1276
[Krauss,Schönherr,Siegert,SH] arXiv:1207.5030

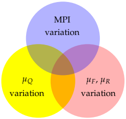


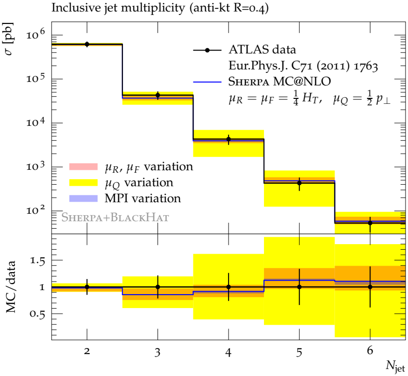


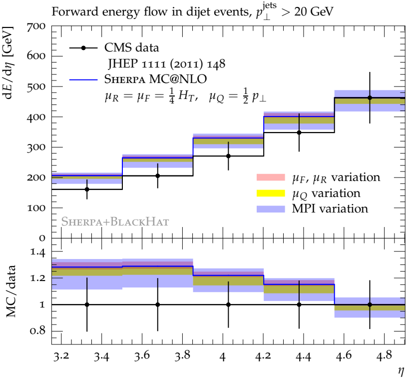
- NLO merging of 0, 1 & 2 jets plus 3 & 4 jets at LO vs MC@NLO merged with up to 4 jets at LO

Practicalities

Scale uncertainties in NLO matching

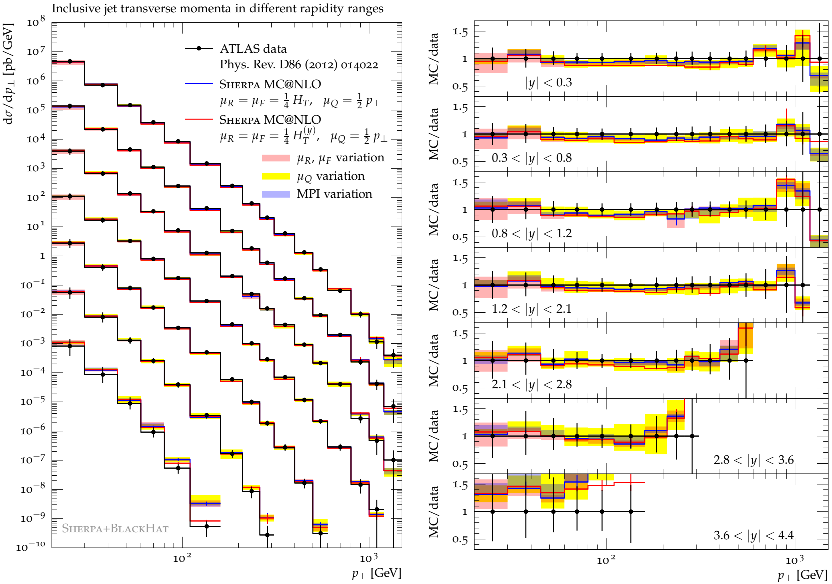






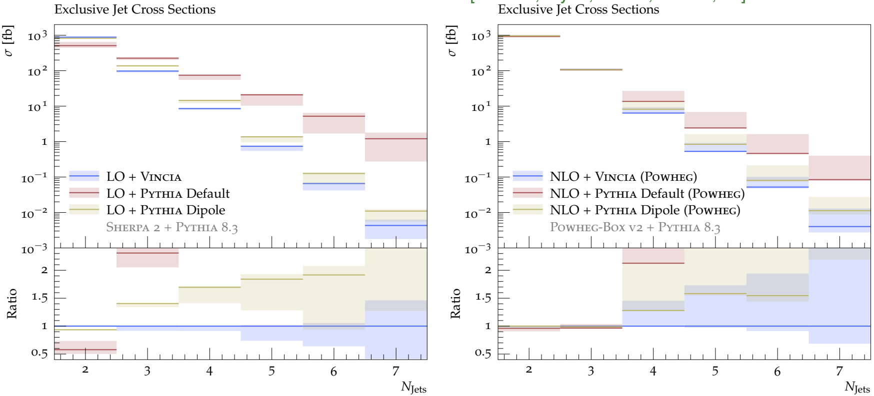
- Jet multiplicity $\rightarrow$ uncertainty due to choice of μ_Q^2
- Forward energy flow $\rightarrow$ major uncertainty from underlying event

Scale uncertainties in NLO matching



PS uncertainties in NLO matching

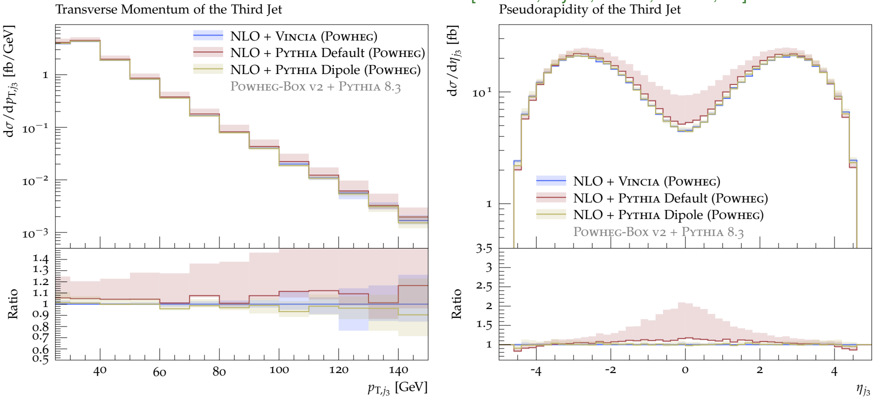
[Mrenna,Payne,Preuss,Skands,SH] arXiv:2106.10987



- LO+PS vs NLO+PS predictions for Pythia variants and Vincia
- Large impact of recoil scheme on sub-leading jet multiplicity

PS uncertainties in NLO matching

[Mrenna,Payne,Preuss,Skands,SH] arXiv:2106.10987

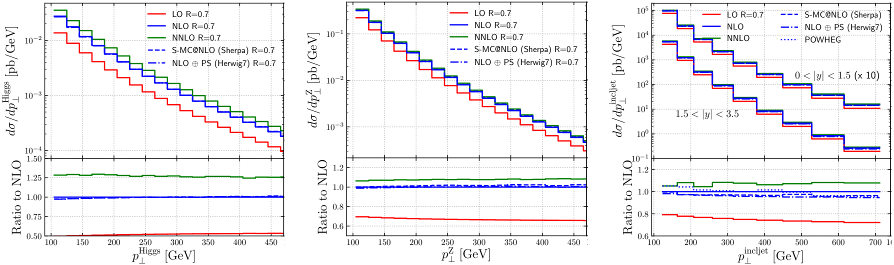


- NLO+PS predictions for Pythia variants and Vincia
- Sizable impact of recoil scheme on sub-leading jet distributions

PS uncertainties in NLO matching

[Bellm at al.] arXiv:1903.12563

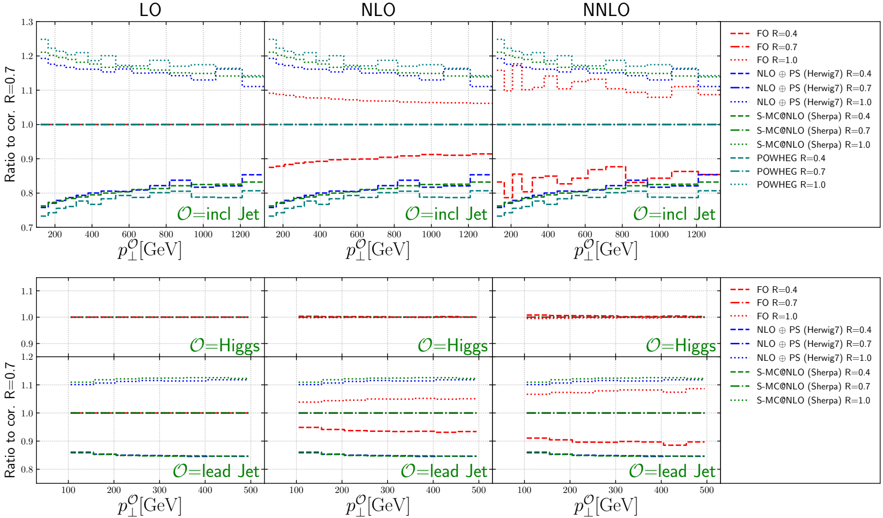
- Ratio of inclusive jet-$p_\perp$ cross sections for different radii in $pp \to jets$



PS uncertainties in NLO matching

[Bellm at al.] arXiv:1903.12563

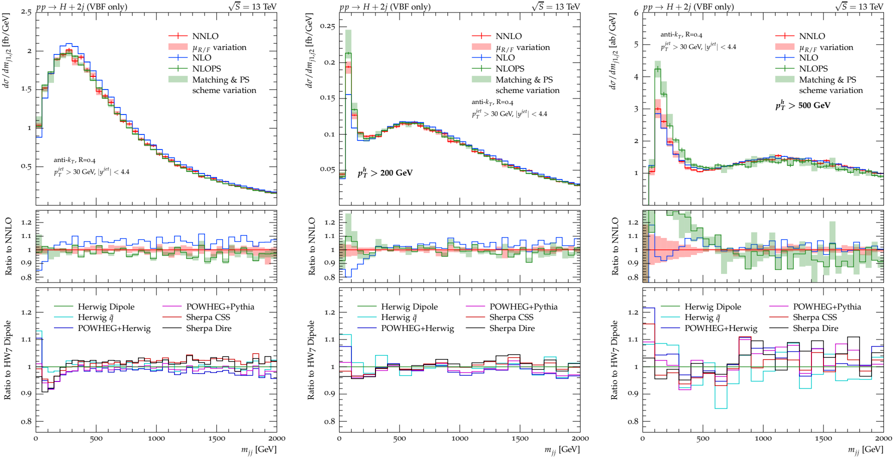
- Ratio of inclusive jet-$p_\perp$ spectra for different radii in $pp \to jj$ / $pp \to H + j$



PS uncertainties in NLO matching

[Buckley et al.] arXiv:2105.11399

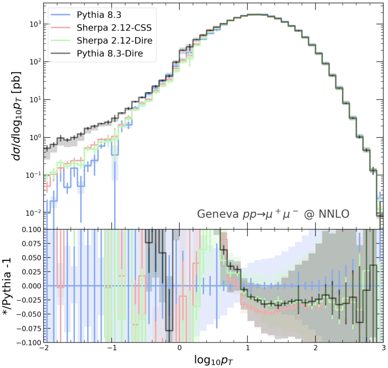
- m_{jj} of two leading jets in VBF Higgs production

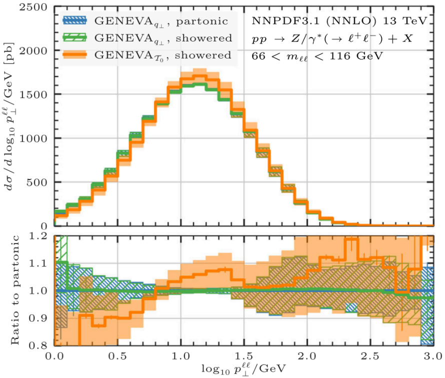


PS uncertainties in NNLO matching

[D. Napoletano, HP2 2022], [Alioli et al.] arXiv:2102.08390

- NNLO+PS precise predictions for $pp \to Z$ from Geneva
- Matched to shower by vetoing events with $r_N(\Phi_{N+M}) > r_N$

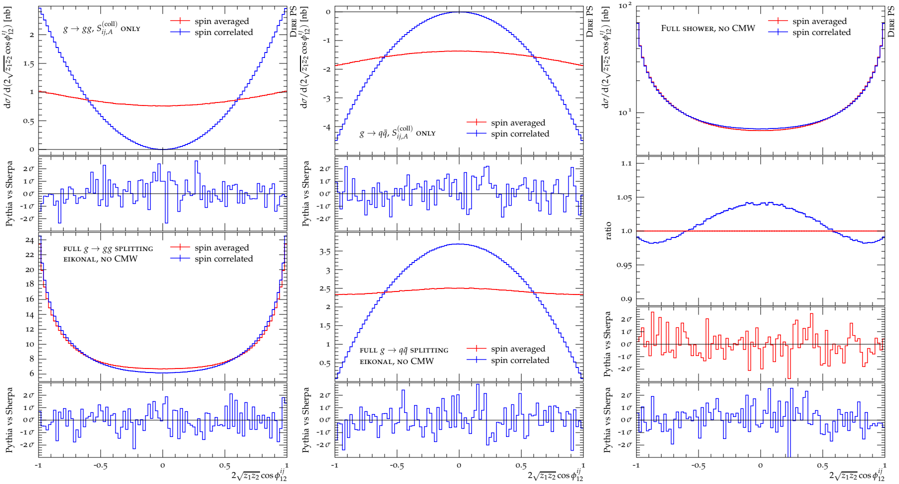




- Parton shower scheme uncertainty
- Choice of resolution variable

Impact of spin correlations

[Dulat,Prestel,SH] arXiv:1805.03757



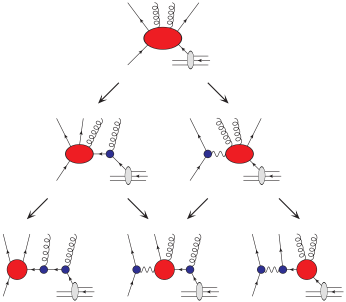
- Spin effects at $\mathcal{O}(\alpha_s^2)$ from double-soft / triple-collinear radiation pattern
- Is overall impact larger than QCD uncertainties?

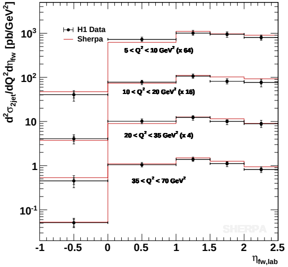
Lessons from HERA

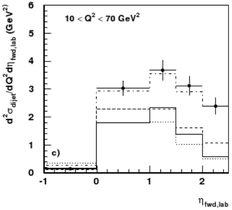
[Carli,Gehrmann,SH] arXiv:0912.3715

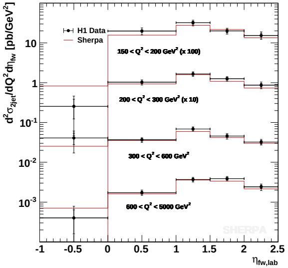
Simulation often too focused on resonant contributions

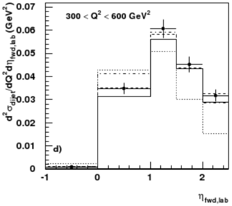
Need be inclusive to describe DIS, low-mass Drell-Yan or photon / diphoton production











Unitarization

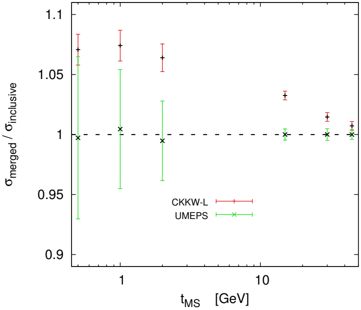
[Lönnblad,Prestel] arXiv:1211.4827, [Plätzer] arXiv:1211.5467
[Bellm,Gieseke,Plätzer] arXiv:1705.06700

- Unitarity condition of PS:

$$1 = \Delta^{(\mathrm{K})}(t_c) + \int_{t_c} \mathrm{d}\Phi_1\, \mathrm{K}(\Phi_1)\, \Delta^{(\mathrm{K})}(t)$$

- ME+PS(@NLO) violates PS unitarity as ME ratio replaces splitting kernels in emission terms, but not in Sudakovs

$$\mathrm{K}(\Phi_1) \to \frac{\mathrm{R}(\Phi_1, \Phi_B)}{\mathrm{B}(\Phi_B)}$$



- Can be corrected by explicit subtraction

$$1 = \underbrace{\left\{ \Delta^{(\mathrm{K})}(t_c) + \int_{t_c} \mathrm{d}\Phi_1 \left[\mathrm{K}(\Phi_1) - \frac{\mathrm{R}(\Phi_1, \Phi_B)}{\mathrm{B}(\Phi_B)} \right] \Theta(Q - Q_{\mathrm{cut}})\, \Delta^{(\mathrm{K})}(t) \right\}}_{\text{unresolved emission / virtual correction}}$$

$$+ \underbrace{\int_{t_c} \mathrm{d}\Phi_1 \left[\mathrm{K}(\Phi_1)\Theta(Q_{\mathrm{cut}} - Q) + \frac{\mathrm{R}(\Phi_1, \Phi_B)}{\mathrm{B}(\Phi_B)} \Theta(Q - Q_{\mathrm{cut}}) \right] \Delta^{(\mathrm{K})}(t)}_{\text{resolved emission}}$$

Summary of lectures

- Matching and merging needed to solve problems
  with both parton showers and fixed-order QCD
- NLO matching and merging de-facto standard at LHC
- Moving towards NNLO accurate matching for HL-LHC
- Making correct predictions not always straightforward
  NLO just a label, getting physics right requires thought

Backup Slides

Collinear parton evolution

- DGLAP equation for fragmentation functions

$$\frac{\mathrm{d}\, xD_a(x,t)}{\mathrm{d}\ln t} = \sum_{b=q,g} \int_0^1 \mathrm{d}\tau \int_0^1 \mathrm{d}z\, \frac{\alpha_s}{2\pi} \big[zP_{ab}(z)\big]_+ \tau D_b(\tau,t)\, \delta(x-\tau z)$$

- Refine plus prescription $\big[zP_{ab}(z)\big]_+ = \lim_{\varepsilon\to 0} zP_{ab}(z,\varepsilon)$

$$P_{ab}(z,\varepsilon) = P_{ab}(z)\,\Theta(1-\varepsilon-z) - \delta_{ab}\frac{\Theta(z-1+\varepsilon)}{\varepsilon} \sum_{c\in\{q,g\}} \int_0^{1-\varepsilon} \mathrm{d}\zeta\, \zeta\, P_{ac}(\zeta)$$

- Rewrite for finite ε

$$\frac{\mathrm{d}\ln D_a(x,t)}{\mathrm{d}\ln t} = -\sum_{c=q,g} \int\limits_0^{1-\varepsilon} \mathrm{d}\zeta\, \zeta\, \frac{\alpha_s}{2\pi} P_{ac}(\zeta) + \sum_{b=q,g} \int\limits_x^{1-\varepsilon} \frac{\mathrm{d}z}{z} \frac{\alpha_s}{2\pi} P_{ab}(z) \frac{D_b(x/z,t)}{D_a(x,t)}$$

- First term is derivative of Sudakov factor $\Delta = \exp\{-\lambda\}$

$$\Delta_a(t,Q^2) = \exp\left\{ -\int_t^{Q^2} \frac{\mathrm{d}\bar{t}}{\bar{t}} \sum_{c=q,g} \int_0^{1-\varepsilon} \mathrm{d}\zeta\, \zeta\, \frac{\alpha_s}{2\pi} P_{ac}(\zeta) \right\}$$

Above equation is parton-shower equivalent of DGLAP

Collinear parton evolution

- At any order in perturbation theory, splitting functions obey sum rules

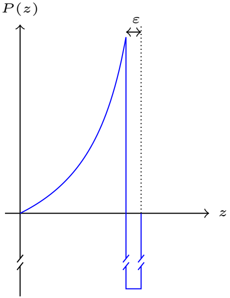
$$\int_0^1 \mathrm{d}\zeta\, \hat{P}_{qq}(\zeta) = 0 \qquad \rightarrow \qquad \text{flavor sum rule}$$

$$\sum_{c=q,g} \int_0^1 \mathrm{d}\zeta\, \zeta\, \hat{P}_{ac}(\zeta) = 0 \qquad \rightarrow \qquad \text{momentum sum rule}$$

$\rightarrow$ defines regularized splitting functions $\hat{P}_{ab}$ as ($\nearrow$ previous slide)

$$\hat{P}_{ab}(z) = \lim_{\varepsilon\to 0}\left[P_{ab}(z)\Theta(1-\varepsilon-z) - \delta_{ab}\frac{\Theta(z-1+\varepsilon)}{\varepsilon}\sum_{c=q,g}\int_0^{1-\varepsilon} d\zeta\,\zeta\,P_{ac}(\zeta)\right]$$

- **What does that mean in physics terms?**
  - Contribution $\propto \Theta(1-\varepsilon-z)$ corresponds to real-emission correction
  - Contribution $\propto \Theta(z-1+\varepsilon)$ corresponds to approximate virtual correction
  - **Momentum sum rule is a unitarity constraint Parton showers implement this automatically**



Truncated unvetoed parton showers

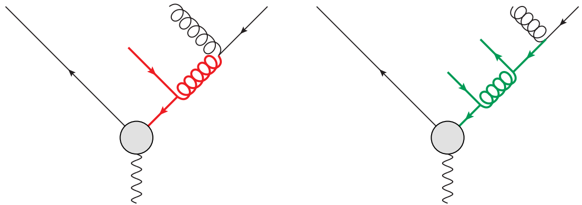
[Nason] hep-ph/0409146

- For $t \neq Q$, PS may generate emissions between μ_Q^2 and $t(\Phi_R)$, as

$$\Delta(t,\mu_Q^2) = \Delta(t,\mu_Q^2;{>}Q_{\rm cut})\,\Delta(t,\mu_Q^2;{<}Q_{\rm cut})$$

$$\Delta(t,\mu_Q^2;{>}Q_{\rm cut}) = \exp\left\{-\int_t^{\mu_Q^2} {\rm d}\Phi_1\,K(\Phi_1)\,\Theta(Q-Q_{cut})\right\}$$

- Momentum and flavor conserving implementation non-trivial
  Example: Two emissions may be allowed, while one may be not

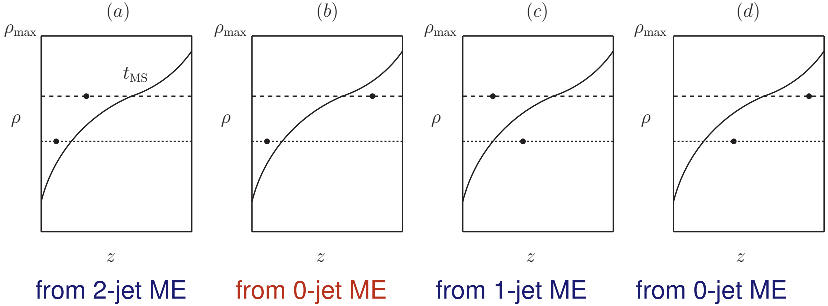


- Effects of non-trivial terms formally suppressed
  Better algorithm may be easier to implement

Circumventing truncated unvetoed parton showers

[Lönnblad] hep-ph/0112284

- Generate truncated unvetoed configurations with parton shower effective redefinition of Q, assuming PS ordering parameter $\sim$ "hardness"
- Schematic illustration of phase space coverage



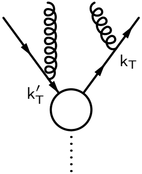
- Straightforward implementation, no reshuffling of kinematics or flavor

A different perspective on NLO merging

- Define compound evolution kernel

$$\tilde{\mathrm{K}}_k(\Phi_{k+1}) = \mathrm{K}_k(\Phi_{k+1})\,\Theta(t_k - t_{k+1}) + \sum_{i=n}^{k-1} \mathrm{K}_i(\Phi_i)\,\Theta(t_i - t_{k+1})\Theta(t_{k+1} - t_{i+1})$$

- Extend modified subtraction



$$\tilde{\mathrm{B}}_k^{(\mathrm{K})}(\Phi_k) = \Big[\mathrm{B}_k(\Phi_k) + \tilde{\mathrm{V}}_k(\Phi_k) + \mathrm{I}_k(\Phi_k)\Big] + \int \mathrm{d}\Phi_1 \Big[\mathrm{B}_k(\Phi_k)\tilde{\mathrm{K}}_k(\Phi_1) - \mathrm{S}_k(\Phi_{k+1})\Big]$$

$$\tilde{\mathrm{H}}_k^{(\mathrm{K})}(\Phi_{k+1}) = \mathrm{R}_k(\Phi_{k+1}) - \mathrm{B}_k(\Phi_k)\tilde{\mathrm{K}}_k(\Phi_1)$$

- Differential event rate for exclusive $n+k$-jet events

$$\langle O\rangle_k^{\mathrm{excl}} = \int \mathrm{d}\Phi_k\, \tilde{\mathrm{B}}_k^{(\mathrm{D})}\,\Theta(Q_k - Q_{\mathrm{cut}}) \times \left[\tilde{\Delta}_k^{(\mathrm{K})}(t_c,\mu_Q^2)\,O_k + \int_{t_c}^{\mu_Q^2} \mathrm{d}\Phi_1\, \tilde{\mathrm{K}}_k\, \tilde{\Delta}_k^{(\mathrm{K})}(t,\mu_Q^2)\,\Theta(Q_{\mathrm{cut}} - Q_{k+1})\,O_{k+1}\right] + \int \mathrm{d}\Phi_{k+1}\, \tilde{\mathrm{H}}_k^{(\mathrm{D})}\, \tilde{\Delta}_k^{(\mathrm{K})}(t_{k+1},\mu_Q^2)\,\Theta(Q_{\mathrm{cut}} - Q_{k+1})$$